A Note on Lavi, Mu’alem, and Nisan (2009, SCF)

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In this short note we correct a minor mistake in the first proof of Lavi, Mu’alem and Nisan (2009, SCF). To be specific, the authors mistakenly take $C = C(x, y)$ instead of $\hat{C}(x, y)$ in the proof of Claim 6. In particular, they reason that $\alpha/2 \notin C$ implies $\alpha/2 + \gamma(x, y) \cdot \vec{l} \notin \hat{P}(x, y)$ in the second line on p. 418. However, this is incorrect because one can only conclude that $\alpha/2 + \gamma(x, y) \cdot \vec{l} \notin \hat{P}(x, y)$; it is still possible that $\alpha/2 + \gamma(x, y) \cdot \vec{l}$ belongs to the boundary of $P(x, y)$, and hence also belongs to $P(x, y)$. In the following, we will stick to the notations of the paper and provide a correct proof of this claim.

Proof (of Claim 6): The first part of the proof is correct; thus we only need to show that $\forall \alpha \in C$, we have $\alpha/2 \in C$ as well. Assume $\alpha \in C$, but $\alpha/2 \notin C$. Then, by the definition of interior and Claim 5, we have

$$\exists \varepsilon > 0, \text{ s.t. } \alpha - \varepsilon \in C(x, y),$$

$$\iff \exists \varepsilon > 0, \text{ s.t. } \alpha - \varepsilon + \gamma(x, y) \cdot \vec{l} \in \hat{P}(x, y) \quad (1)$$

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1Claim 6 establishes that set $C$ is convex, see p.417-18.
2In the paper, the interior of set $C$ is defined as $\hat{C} = \{\alpha \in C | \alpha - \varepsilon \in C \text{ for some } \varepsilon > 0\}$, see line 12 on p. 417. This definition is not standard, but, with the other properties of set $C$, is equivalent to the standard definition.
Moreover,

\[
\forall \varepsilon > 0, \quad \frac{\alpha}{2} - \varepsilon \notin C(z, y),
\]

\[
\iff \forall \varepsilon > 0, \quad \frac{\alpha}{2} - \varepsilon + \gamma(z, y) \cdot \vec{1} \notin P(z, y),
\]

\[
\implies \forall \varepsilon > 0, \quad -\frac{\alpha}{2} + \varepsilon + \gamma(y, z) \cdot \vec{1} \in P(y, z),
\]

where the second implication follows Claim 2 and 4. In particular, we can pick \( \varepsilon = \varepsilon/4 \).

Now, (2) becomes

\[
\left( -\frac{\alpha}{2} + \frac{\varepsilon}{2} \right) - \frac{\varepsilon}{4} + \gamma(y, z) \cdot \vec{1} \in P(y, z).
\]

(3)

Summing up (1) and (3) and applying Claim 3 and Claim 4, one obtains

\[
\left( \alpha - \frac{\alpha}{2} + \frac{\varepsilon}{2} \right) - (\varepsilon + \frac{\varepsilon}{4})/2 + \gamma(x, z) \cdot \vec{1} \in P(x, z)
\]

\[
\iff \frac{\alpha}{2} - \frac{\varepsilon}{8} + \gamma(x, z) \cdot \vec{1} \in P(x, z)
\]

\[
\iff \frac{\alpha}{2} - \frac{\varepsilon}{8} \in C(x, z)
\]

In other words, there exists \( \varepsilon' = \varepsilon/8 > 0 \), s.t. \( \alpha/2 - \varepsilon' \in C(x, z) \). Hence, by the definition of interior and Claim 5, we have

\[
\frac{\alpha}{2} \in \hat{C}(x, z) = C.
\]

We reach a contradiction. Hence, set \( C \) is indeed convex.

References