Conditional Equilibrium Outcomes via Ascending Price Processes with Applications to Combinatorial Auctions with Item Bidding

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A Walrasian equilibrium in an economy with non-identical indivisible items exists only for small classes of players' valuations (mostly "gross substitutes" valuations), and may not generally exist even with decreasing marginal values. This paper studies a relaxed notion, "conditional equilibrium", that requires individual rationality and "outward stability", i.e., a player will not want to *add* items to her allocation, at given prices. While a Walrasian equilibrium outcome is unconditionally stable, a conditional equilibrium outcome is stable if players cannot choose to drop only *some* of their allocated items.

With decreasing marginal valuations, conditional equilibrium outcomes exhibit three appealing properties: (1) An approximate version of the first welfare theorem, namely that the social welfare in any conditional equilibrium is at least half of the maximal welfare; (2) A conditional equilibrium outcome can always be obtained via a natural ascending-price process; and (3) The second welfare theorem holds: any welfare maximizing allocation is supported by a conditional equilibrium. In particular, each of the last two properties independently implies that a conditional equilibrium always exists with decreasing marginal valuations (whereas a Walrasian equilibrium generally does not exist for this common valuation class). Additional strategic foundation is provided via a strong connection to Nash equilibria of combinatorial auctions with item bidding, enabling us to strengthen results of Bhawalkar and Roughgarden (SODA'11) for this auction game.

Given these appealing properties we ask what is a maximal valuation class that ensures the existence of a conditional equilibrium and includes unit-demand valuations. Our main technical results provide upper and lower bounds on such a class. The lower bound shows that there exists such a class that is significantly larger than gross-substitutes, and that even allows for some (limited) mixture of substitutes and complements. For three items or less our bounds are tight, implying that we completely identify the unique such class. The existence proofs are constructive, and use a "flexible-ascent" auction that is based on algorithms previously suggested for "fractionally subadditive" valuations. This auction is slightly different from standard ascending auctions, as players may also decrease prices of obtained items in every iteration, as long as their overall price strictly increases.

Categories and Subject Descriptors: J.4 [**Social and Behavioral Sciences**]: Economics

General Terms: Economics, Theory

Additional Key Words and Phrases: Walrasian Equilibrium, Ascending auctions

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This work is supported by a grant from the USA-Israel Bi-national Science Foundation. Hu Fu was also supported by NSF grants CCF-0643934 and AF-0910940. Robert Kleinberg was also partially supported by NSF awards CCF-0643934 and AF-0910940, an Alfred P. Sloan Foundation Fellowship, a Microsoft Research New Faculty Fellowship, and a Google Research Grant. Ron Lavi was also supported by a grant from the Israeli Ministry of Science and by the Google inter-university center for electronic markets and auctions.
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EC'12, June 4–8, 2012, Valencia, Spain.
ACM 978-1-4503-1415-2/12/06.