Exercise 1 – Auction Theory (96573)

1. Give a formal proof that truthfulness in the second-price auction for n players is a dominant strategy.

2. Find a Bayesian-Nash equilibrium in the first-price auction, when players’ values are independently drawn from the uniform distribution on \([a,b]\), for any \(b > a \geq 0\). Hints for one possible solution:
   a. Assume that the equilibrium bid function is \(b(z) = \alpha + \beta z\), and that \(b(a) = a\).
   b. Start by writing the utility function \(u(x,z)\) that denotes a player’s utility when her value is \(x\) and she bids \(b(z)\), for arbitrary \(x,z\).
   c. Since \(b(z)\) is an equilibrium, it follows that a player’s utility is maximized when she bids \(b(x)\). This should give you a first-order condition on the bid function that will lead you to get an exact expression for \(\alpha\) and \(\beta\), which gives you the bid function.

3. In the setting of the previous question, suppose there are 3 players with values independently drawn from the uniform distribution on \([5,10]\). Suppose player 1 has value \(v_1 = 8\). What is her equilibrium bid? Show that she will not improve her expected utility by declaring 6 or by declaring 8.