

# Blocked Clause Elimination

## Blocked Clauses [Kullmann'99]

### Definition (Blocking literal)

A literal  $l$  in a clause  $C$  of a CNF  $F$  blocks  $C$  w.r.t.  $F$  if for every clause  $D \in F_{\bar{l}}$ , the resolvent  $(C \setminus \{l\}) \cup (D \setminus \{\bar{l}\})$  obtained from resolving  $C$  and  $D$  on  $l$  is a tautology.

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### Example

Consider the formula  $(a \vee b) \wedge (a \vee \bar{b} \vee \bar{c}) \wedge (\bar{a} \vee c)$ .

First clause is not blocked.

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### Proposition

Removal of an arbitrary blocked clause preserves satisfiability.

# Blocked Clause Elimination (BCE)

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While there is a blocked clause  $C$  in a CNF  $F$ , remove  $C$  from  $F$ .

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## Proposition

BCE is confluent, i.e., has a unique fixpoint

- ▶ Blocked clauses stay blocked w.r.t. removal

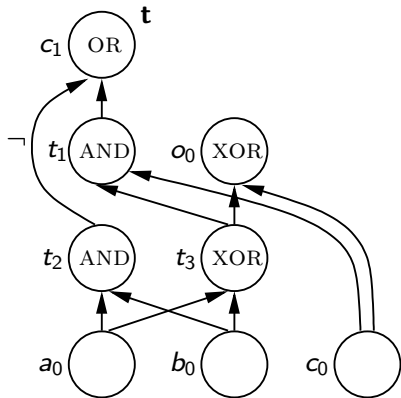


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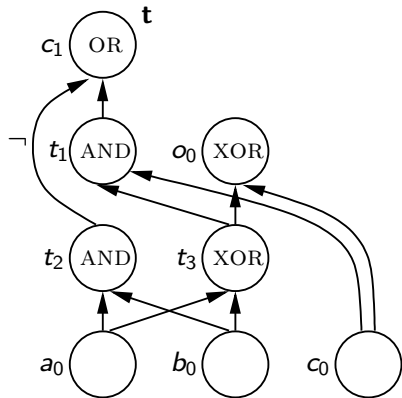


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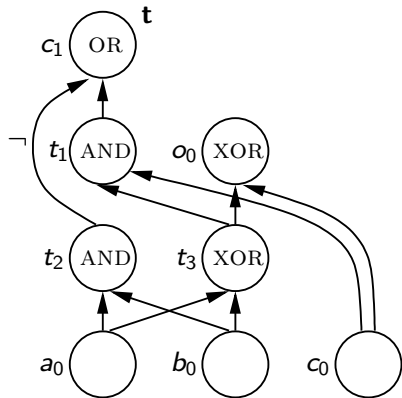


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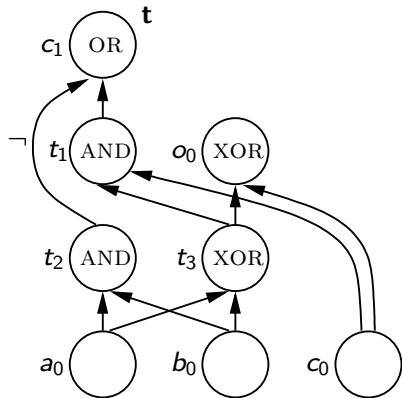


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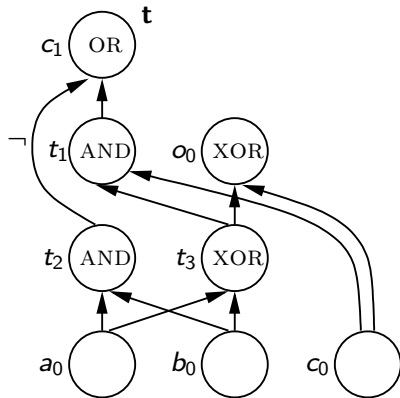


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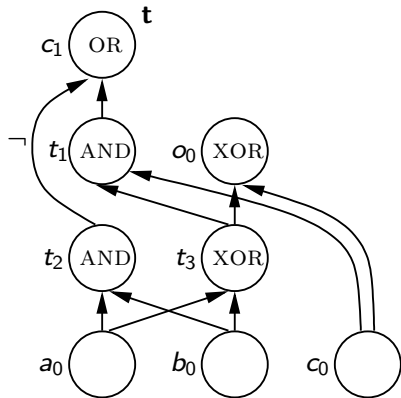


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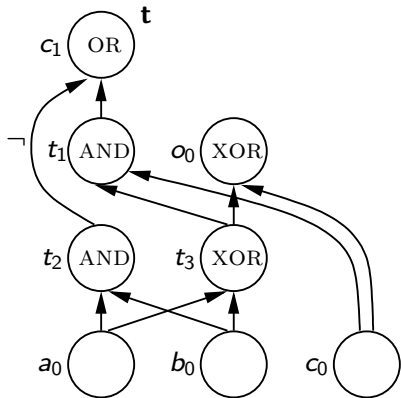
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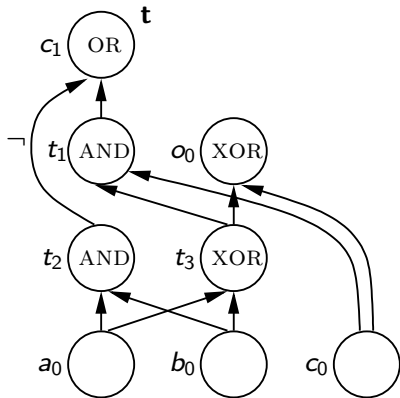


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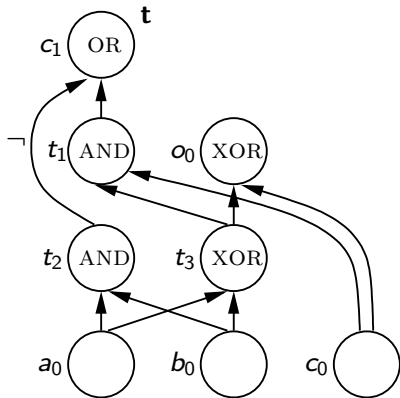
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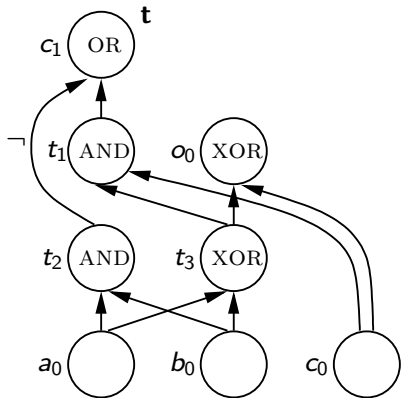


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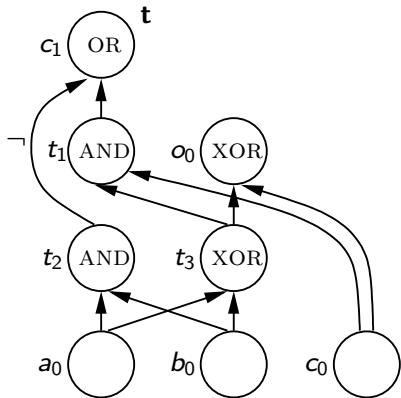


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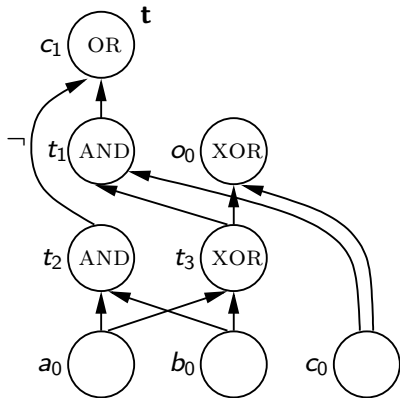


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# Hyper Binary Resolution