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Understanding, improving and parallelizing MUS finding using model rotation

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Minimal Unsatisfiable

- Finding Minimal Unsatisfiable Subsets (MUSes) in unsatisfiable CNF formulas
- An unsatisfiable formula $\mathcal{F}$ is minimal unsatisfiable iff any proper subset of its clauses $\mathcal{F}' \subset \mathcal{F}$ is satisfiable.
- An **assoc** for a clause $c \in \mathcal{F}$ is a complete assignment that satisfies all clauses in $\mathcal{F}$ except c.
- A formula $\mathcal{F}$ is minimal unsatisfiable iff every clause in the formula has at least one assoc.

Classical destructive algorithm

1. $M = \emptyset$
2. `while` $\mathcal{F} \neq M$
3. `pick` a clause $c \in \mathcal{F} \setminus M$
4. `if` $\mathcal{F} \setminus \{c\}$ is satisfiable `then`
5. $M = M \cup \{c\}$
6. `else`
7. $\mathcal{F} = \mathcal{F} \setminus \{c\}$
8. `return` $\mathcal{F}$

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7. $\mathcal{F} = \mathcal{F}'$ s.t. $\mathcal{F}'$ is unsatisfiable and $\mathcal{F}' \subseteq \mathcal{F} \setminus \{c\}$
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$\Rightarrow$ `modelRotate`(c, a) `//(`a`=sat. assign. for` $\mathcal{F} \setminus \{c\}$`)`

6. `else`
7. $\mathcal{F} = \mathcal{F}'$ s.t. $\mathcal{F}'$ is unsatisfiable and $\mathcal{F}' \subseteq \mathcal{F} \setminus \{c\}$
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Model rotation

Marques-Silva and Lynce, SAT2011

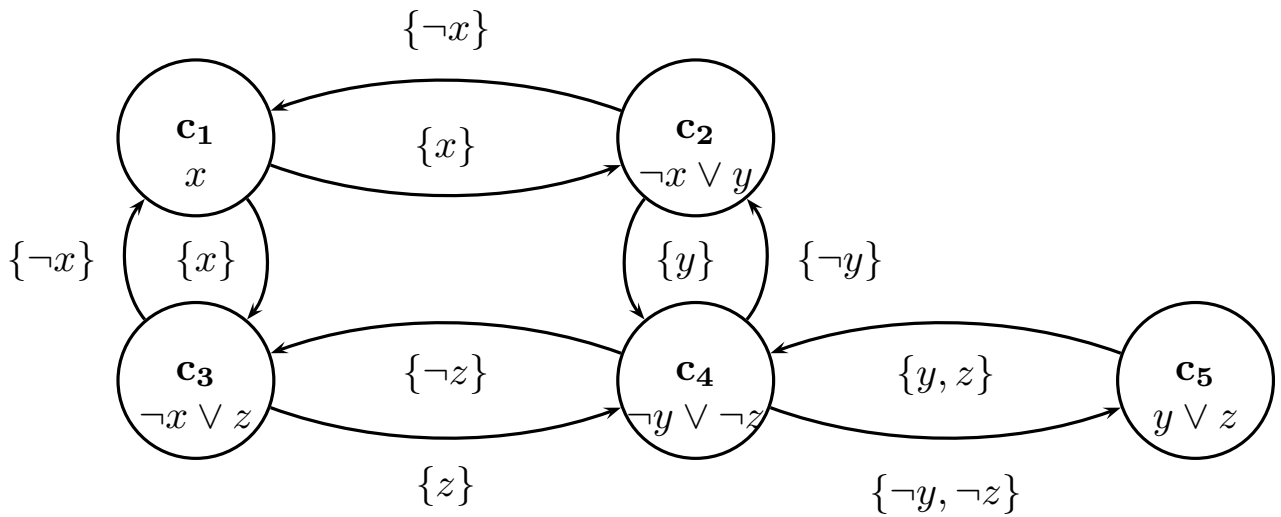
Belov and Marques-Silva, FMCAD2011

function `modelRotate(clause` c`, assignment` a`)`

1. `for all` $l \in c$ `do`
2. $a' = a$ except l is assigned **true** instead of **false**
3. `if` $\left(\begin{array}{l} \text{exactly one clause } c' \in \mathcal{F} \text{ is} \\ \text{not satisfied by } a' \text{ and } c' \notin M \end{array} \right)$ `then`
4. $M = M \cup \{c'\}$
5. `modelRotate`(c', a')

Flip graph

- A vertex for every clause
- Edges are labelled: $L(c_i, c_j) = \{l \mid l \in c_i \text{ and } \neg l \in c_j\}$
- An edge between c_i and c_j iff $L(c_i, c_j) \neq \emptyset$



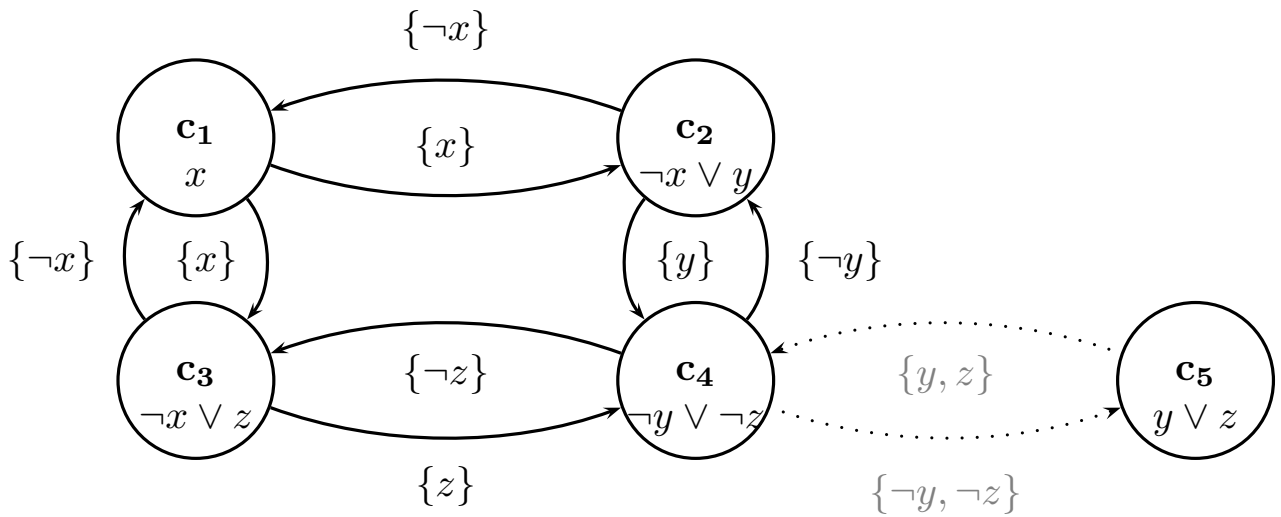
Flip graph

- Lemma 1:

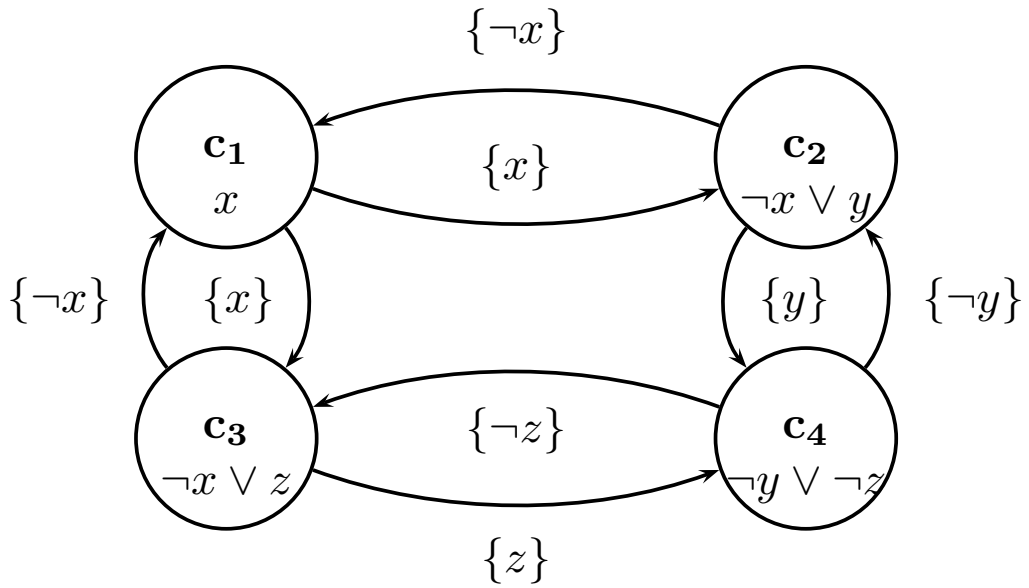
  *Rotating literal l in an assoc for a clause c_i can **not** result in an assoc for clause c_j if $L(c_i, c_j) \neq \{\neg l\}$*

- Possible rotation edges:

  All edges (c_i, c_j) for which $|L(c_i, c_j)| = 1$

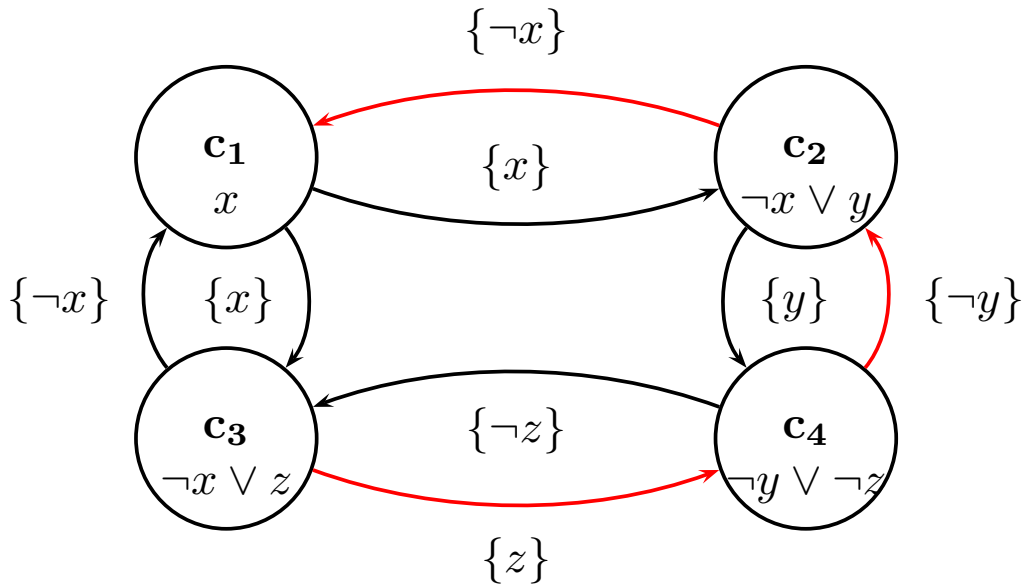


Rotation example



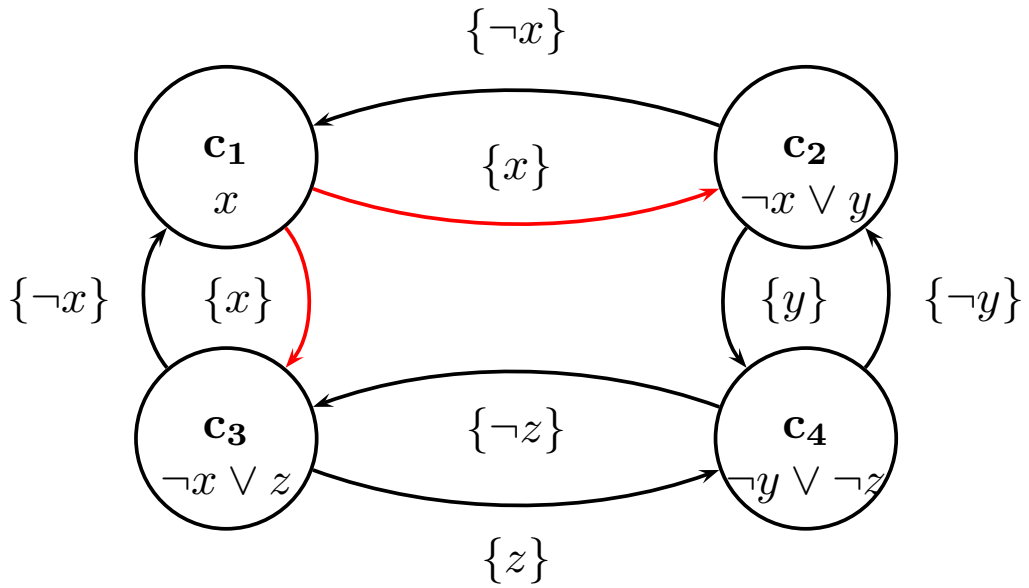
assoc for: c_3 $x = \textbf{true}$ $y = \textbf{true}$ $z = \textbf{false}$

Rotation example



assoc for: c_3	$x =$ **true**	$y =$ **true**	$z =$ **false**
assoc for: c_4	$x =$ **true**	$y =$ **true**	$z =$ **true**
assoc for: c_2	$x =$ **true**	$y =$ **false**	$z =$ **true**
assoc for: c_1	$x =$ **false**	$y =$ **false**	$z =$ **true**

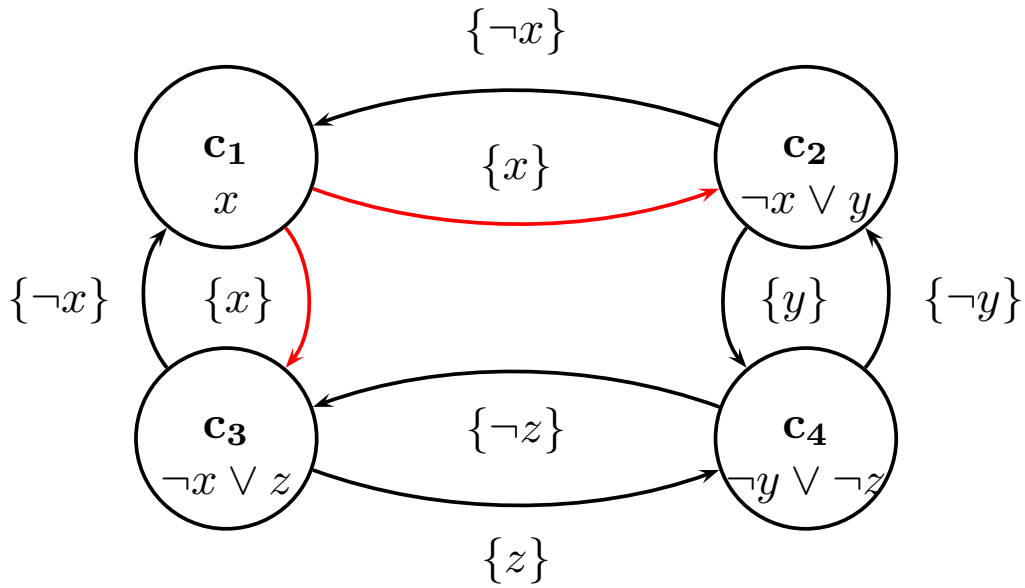
Rotation example



There are 3 possible assocs for c_1.

x=**false** y=**false** z=**true**
x=**false** y=**true** z=**false**
x=**false** y=**false** z=**false**

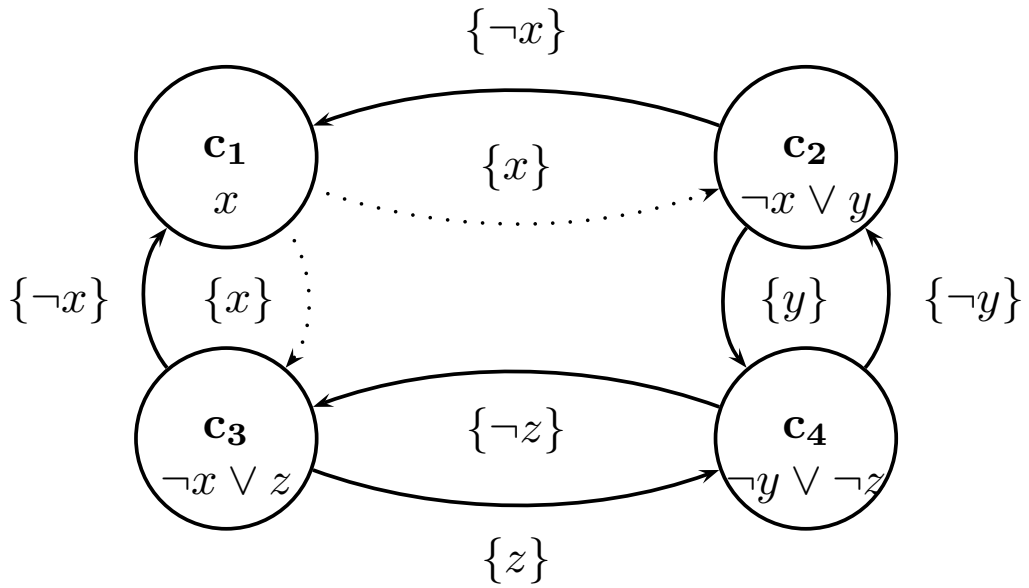
Rotation example



There are 3 possible assocs for c_1. Rotation gives:

x=**true**	y=**false**	z=**true**	$\rightarrow$	assoc for c_2
x=**true**	y=**true**	z=**false**	$\rightarrow$	assoc for c_3
x=**true**	y=**false**	z=**false**	$\rightarrow$	does not satisfy c_2 and c_3!

Rotation example



- Guaranteed rotation edges:

  All possible rotation edges (c_i, c_j) *s.t. for all* $c_k \neq c_j$ *it holds that* $L(c_i, c_j) \neq L(c_i, c_k)$

Rotation theory

- Theorem 1:

  If c_i has an assoc and a guaranteed rotation edge (c_i, c_j) exist then c_j has an assoc

- If c_i has an assoc and a path over guaranteed rotation edges from c_i to c_j exist then c_j has an assoc

- Corollary 1:

  If a path over guaranteed rotation edges between c_i and c_j exists in both directions then c_i has an assoc iff c_j has an assoc

Strongly Connected Components

- A directed graph is *strongly connected* if there exists a path between any two of its vertices
- The *strongly connected components* (SCCs) of a directed graph are its maximal strongly connected subgraphs

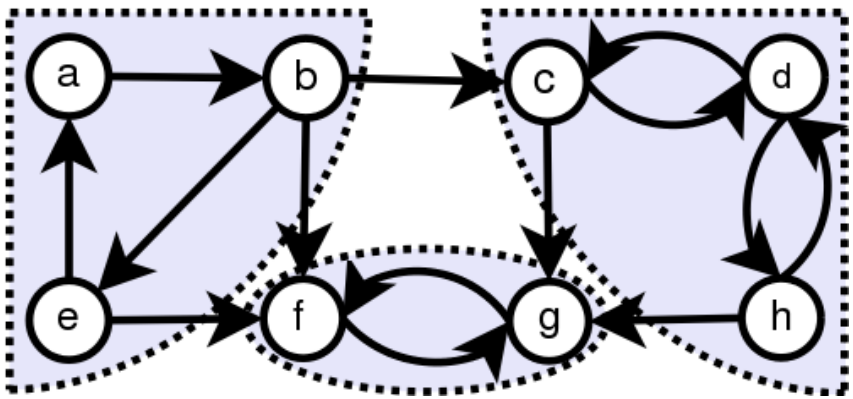


Figure source: `http://en.wikipedia.org/wiki/Strongly_connected_component`

Rotation theory continued

- Let $\mathcal{F}$ be an unsatisfiable formula
- Let E_G be the set of guaranteed rotation edges $\mathcal{F}$ induces
- Consider the SCCs of the graph $G = (\mathcal{F}, E_G)$
- Corollary 2:

  A MUS $\mathcal{F}' \subseteq \mathcal{F}$ can be found using no more SAT solver calls than there are SCCs in $G = (\mathcal{F}, E_G)$

Rotation theory continued

- Let a *root SCC* be an SCC that contains no vertices with incoming edges originating outside the SCC
- Let $\mathcal{F}'$ be an *minimal unsatisfiable* formula, and E'_G the set of guaranteed rotation edges it induces
- Corollary 3:

  An assoc can be found for every clause in $\mathcal{F}'$ using no more SAT solver calls than there are root SCCs in $G' = (\mathcal{F}', E'_G)$

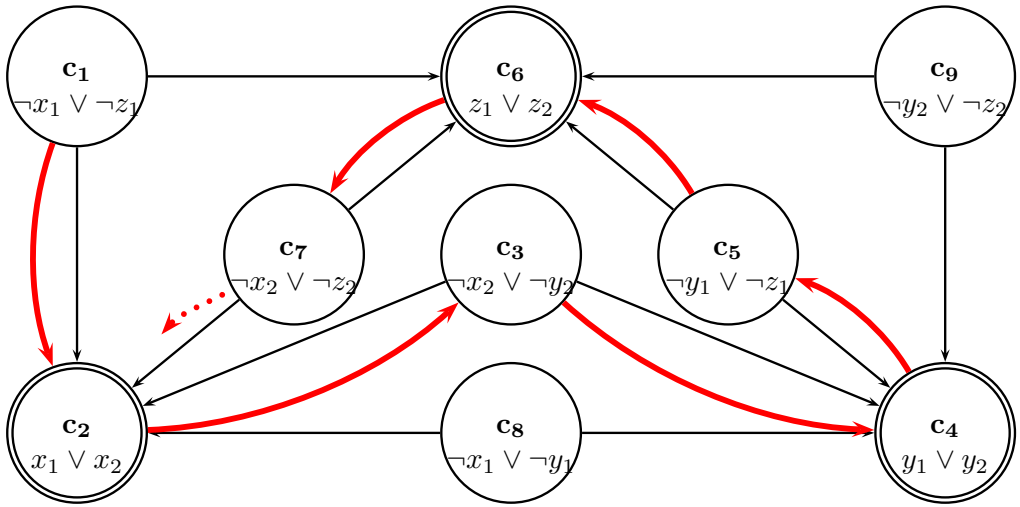
Statistics - benchmarks from Marques-Silva et.al.

	original		MUSes	
# benchmarks	500		491	
# with single root SCC	148		148	
avg. # clauses	6874.4		6204.2	
avg. # SCCs	3000.4	(44%)	2484.1	(40%)
avg. # root SCCs	2124.7		1680.5	(27%)
avg. clause length	2.32		2.35	
avg. out-degree E_P	12.30		11.24	
avg. out-degree E_G	1.60		1.63	

Statistics - benchmarks from SAT11 competition

	original		MUSes	
# benchmarks	298		262	
# with single root SCC	0		51	
avg. # clauses	404574		8162.7	
avg. # SCCs	327815	(81%)	3355.6	(41%)
avg. # root SCCs	258350		1891.1	(23%)
avg. clause length	2.53		2.42	
avg. out-degree E_P	86.84		14.16	
avg. out-degree E_G	0.89		1.59	

Improving & Parallelizing



- The suggested improvement for model rotation concerns cyclic paths.
- The parallelization uses the *Tarmo* parallel incremental SAT solver.

Conclusions

- This work provides new insights on *model rotation*
- The key is to look at the algorithm as a graph search
- Typical benchmarks have properties that guarantee effectiveness of model rotation
- The technique was improved, parallelized and evaluated

- Algorithm 1: Uses standard recursive model rotation.
- Algorithm 2: Uses the SCC graph for choosing clauses c to check in next SAT call

Algorithm 1. MUS finder with recursive model rotation [2,13]

Given an unsatisfiable formula $\mathcal{F}$:

1. $M = \emptyset$
2. `while` $\mathcal{F} \neq M$
3. `pick` a clause $c \in \mathcal{F} \setminus M$
4. `if` $\mathcal{F} \setminus \{c\}$ is satisfiable `then`
5. $M = M \cup \{c\}$
6. `modelRotate`(c, a) where a is a compl. satisfying assign. for $\mathcal{F} \setminus \{c\}$
7. `else`
8. $\mathcal{F} = \mathcal{F} \setminus \{c\}$
9. `return` $\mathcal{F}$

function `modelRotate(clause` c, `assignment` a`)` `// such that` $a \in A(c, \mathcal{F})$

I `for all` $l \in c$ `do`
II $a' =$ `rotate`$(a, \neg l)$
III `if` exactly one clause $c' \in \mathcal{F}$ is not satisfied by a' and $c' \notin M$ `then`
IV $M = M \cup \{c'\}$
V `modelRotate`(c', a')

Algorithm 2. MUS finder respecting upper bounds of Cor. 2 and Cor. 3

Given an unsatisfiable formula $\mathcal{F}$ and the set E_G it induces:

1. Let $S = \{\mathcal{F}_1, \mathcal{F}_2, \cdots, \mathcal{F}_{|S|}\}$ the division of $\mathcal{F}$ into the SCCs of $G = (\mathcal{F}, E_G)$
2. $M = \emptyset$
3. `while` $\mathcal{F} \neq M$
4. `pick` $\mathcal{F}_i \in S$ corresponding to a root SCC such that $M \cap \mathcal{F}_i = \emptyset$
5. `pick` a clause $c \in \mathcal{F}_i$
6. `if` $\mathcal{F} \setminus \{c\}$ is satisfiable `then`
7. $M = M \cup \{c' \mid c' = c$ or c' is reachable from c in $G = (\mathcal{F}, E_G)\}$
8. `else`
9. $\mathcal{F} = \mathcal{F} \setminus \mathcal{F}_i$
10. $S = S \setminus \{\mathcal{F}_i\}$
11. `return` $\mathcal{F}$

Table 3. Instances solved and average solver calls over benchmarks solved by all

Model rotation	none			standard			improved		
	#	SAT	UNSAT	#	SAT	UNSAT	#	SAT	UNSAT
Alg. 1	-	-	-	136	2696.8	193.4	136	2346.1	188.3
Alg. 2	106	4389.2	162.5	123	3610.8	163.6	131	2341.9	172.4
Alg. 1 Tarmo 4	-	-	-	-	-	-	134	2660.9(88%)	436.0(42%)
Alg. 2 Tarmo 4	-	-	-	-	-	-	135	2448.4(95%)	377.2(43%)
Alg. 1 Tarmo 8	-	-	-	-	-	-	136	2963.0(79%)	796.5(23%)
Alg. 2 Tarmo 8	-	-	-	-	-	-	139	2605.7(90%)	700.6(23%)

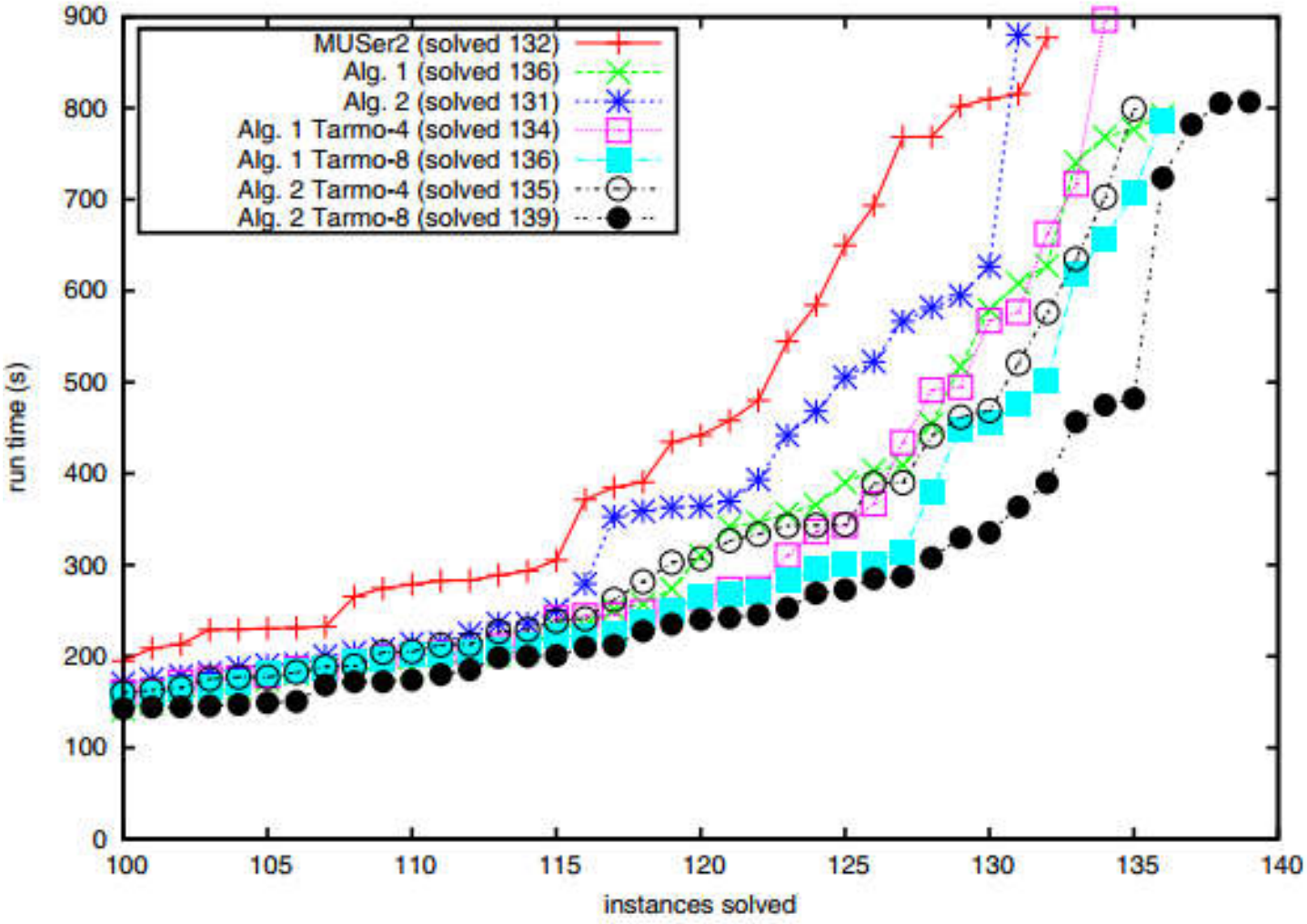


Fig. 3. Comparison of multiple versions

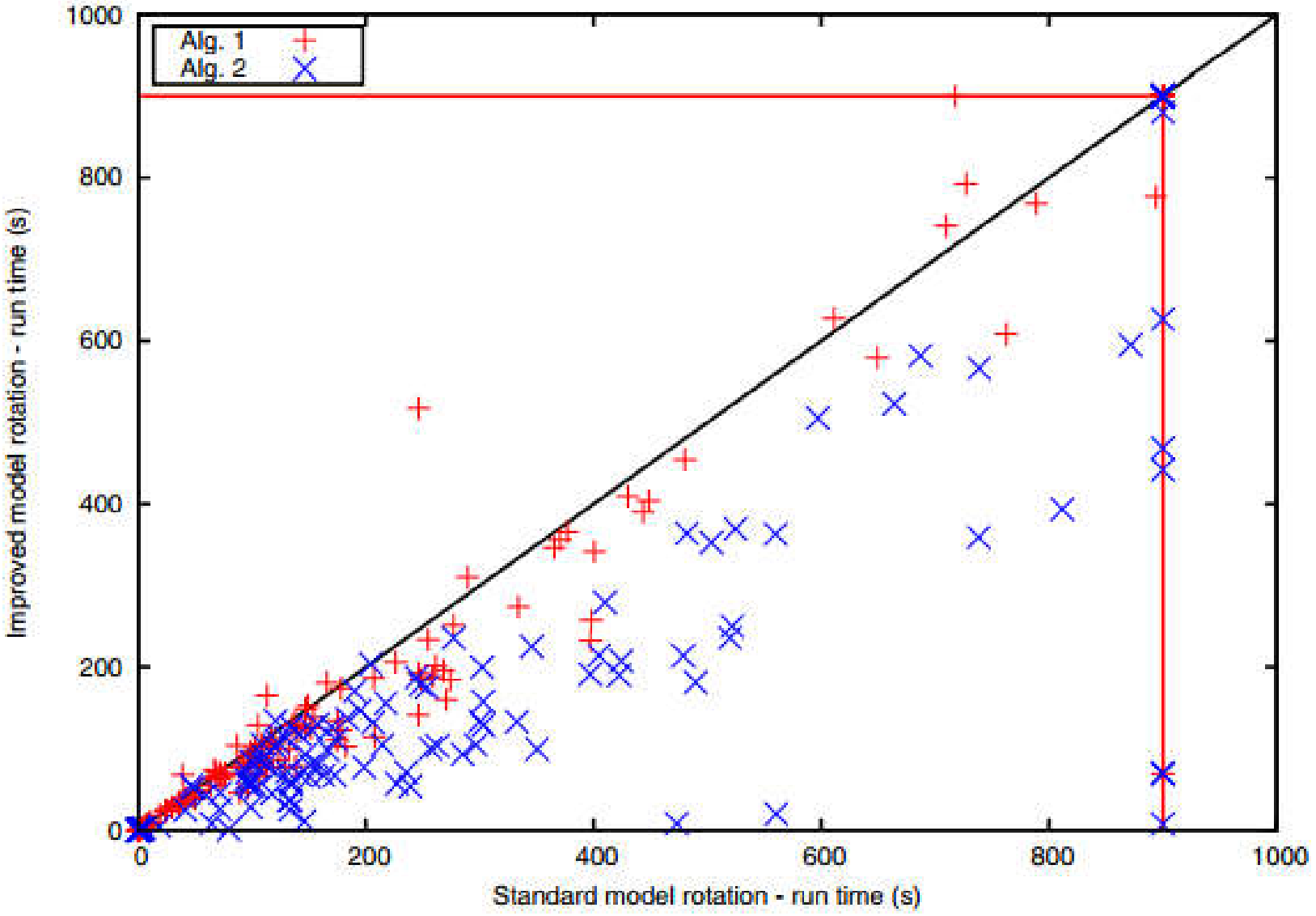


Fig. 4. Effect of improved model rotation

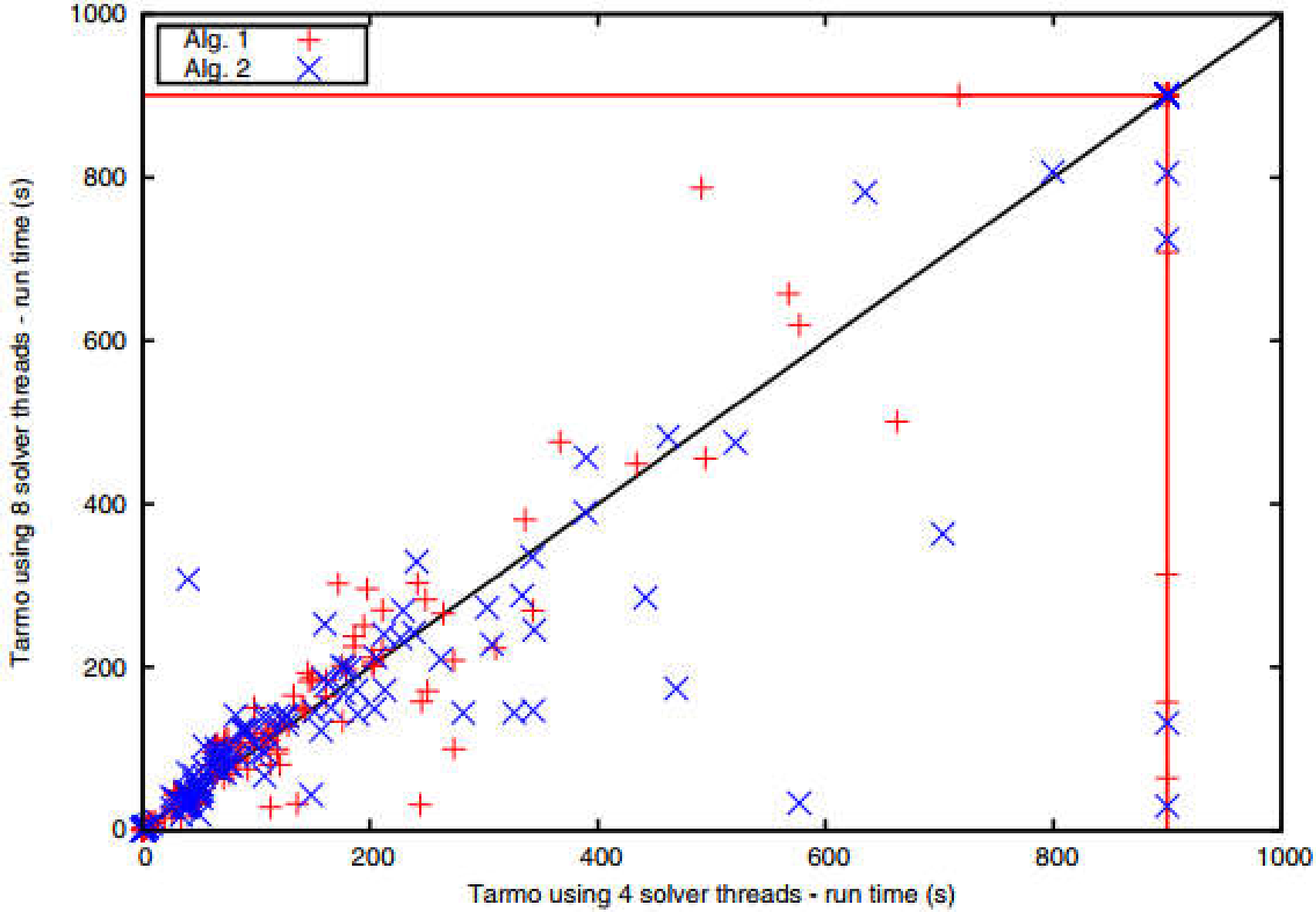


Fig. 5. Performance of 4 versus 8-threads