Time-Bounded Analysis of Real-Time Systems

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Abstract—Real-Time Embedded Software (RTES) constitutes an important sub-class of concurrent safety-critical programs. We consider the problem of verifying functional correctness of periodic RTES, a popular variant of RTES that execute periodic tasks in an order determined by Rate Monotonic Scheduling (RMS). A computational model of a periodic RTES is a finite collection of terminating tasks that arrive periodically and must complete before their next arrival.

We present an approach for time-bounded verification of safety properties in periodic RTES. Our approach is based on sequentialization. Given an RTES $C$ and a time-bound $W$, we construct (and verify) a sequential program $S$ that over-approximates all executions of $C$ up to time $W$, while respecting priorities and bounds on the number of preemptions implied by RMS. Our algorithm supports partial-order reduction, preemption locks, and priority locks. We implemented our approach for C programs, with properties specified via user-provided assertions. We evaluated our tool on several realistic examples, and were able to detect a subtle concurrency issue in a robot controller.

I. INTRODUCTION

Real-Time Embedded Software (RTES) is an important sub-class of concurrent safety-critical programs. They play a crucial role in controlling systems ranging from airplanes and cars, to infusion pumps and microwaves. We are increasingly reliant on these cyber-physical systems to maintain our modern technology-driven way of life. As such, verifying the correct operation of RTES is an important and open challenge. Addressing this challenge is the subject of our paper.

Specifically, we focus on systems that receive as input a collection of periodic tasks, where each task $T_i$ has, among other things, a terminating task body $T_i$ and a period $P_i$. The tasks are prioritized in a Rate-Monotonic [1] fashion, which means that tasks with shorter periods have higher priorities. We call such an input pattern a periodic program.

A periodic program $C$ is executed by running its tasks periodically and concurrently with asynchronous priority-sensitive interleaving. Thus, at each scheduling point, the active task with the highest priority is selected for execution. A task $T_i$ becomes inactive at the end of its body $T_i$, and is reactivated after $P_i$ time has passed since its last activation. A single execution of a task body is called a job. It is convenient to view an execution of $C$ as an asynchronous priority-sensitive interleaving of the jobs statements, where the statements arise from the infinite job streams corresponding to the periodic execution of the task bodies.

Periodic programs constitute an important fragment of RTES that interact with the physical world. In particular, the task periods are dictated by the physical environment and the underlying control algorithms. Consider, for example, the nxt/OSEK-based [2] LEGO MINDSTORM robot controller. It has three periodic tasks: a balancer, with a 4 millisecond (ms) period, maintains the balance of the robot; obstacle, with a 50 ms period, monitors a sonar sensor to detect obstacles; and bluetooth, with a 100 ms period, monitors a bluetooth link for remote commands from the user. Another example is a generic avionic mission system that was described in [3]. It includes 10 periodic tasks, including weapon release (10 ms), radar tracking (40 ms), target tracking (40 ms), aircraft flight data (50 ms), display (50 ms) and steering (80 ms). Other examples of periodic programs include phase-array radars and aircraft collision-avoidance systems.

These examples demonstrate the fact that periodic programs are used for developing a wide range of RTES that interact with the physical world, and play an important role in the correct operation of safety-critical systems. Statically predicting behavior – by verifying logical and timing properties of periodic programs – is a problem of great practical relevance.

Despite a wide body of work, the state-of-the-art in verification of real-time and concurrent programs does not address logical properties of periodic programs (with the exception of the recent work of Kidd et al. [4], which we discuss in Sec. VIII). On one hand, techniques for verifying properties of timed systems [5], [6] are based on Timed Automata [7]. They abstract away significantly the behavior (i.e., control- and data-flow) of target systems, and, therefore, are unsuitable for analyzing logical properties. On the other hand, approaches for concurrent software verification (e.g., [8]) employ a non-deterministic scheduler model (i.e., tasks do not have priorities or periods), and thus cannot handle the execution semantics of periodic programs. Against this backdrop, our main contribution is the development and evaluation of an approach to verify logical properties of periodic programs.

Specifically, we present an approach for time-bounded verification of safety properties of periodic programs. The inputs are: (i) a periodic program $C$; (ii) a safety property expressed via an assertion $A$ embedded in $C$, (iii) an initial condition $Init$ of $C$, and (iv) a time bound $W$. Time-bounded verification can be seen as an analogue of Boundred Model Checking (BMC) for RTES, since time is a natural way to bound an execution of a periodic program for the purpose of verification.

Our solution for the time-bounded verification problem is based on sequentialization – reducing verification of a concurrent program to verification of a sequential program. It is inspired by work on sequentialization for Context-Bounded
Analysis [8], [9], [10], [11], [12] (CBA) and Bounded Model Checking [13] (BMC). A key distinguishing aspect of our work is that instead of bounding the number of context switches (as in CBA), or the number of execution steps (as in BMC), the input time bound \( \mathcal{W} \) translates in our model to a bound on the number of jobs. This is a natural consequence of the fact that tasks are periodic and, therefore, are activated a finite number of times within \( \mathcal{W} \). Our solution also handles two types of locks used commonly in periodic programs, and incorporates two forms of partial-order reduction aimed at reducing analysis time.

We have implemented our solution in a tool, called REk. Our tool is able to verify periodic programs implemented in C where tasks communicate via shared variables and synchronize via locks. We have used it to verify several variants of a nxt/OSEK-based [2] LEGO MINDSTORM robot controller. In some instances, we found subtle concurrency issues in the controller using our tool. We have also evaluated our tool on several custom versions of the reader-writer protocol.

The rest of the paper is structured as follows: in the next section we formally define a periodic program and its semantics. In Sec. III, we describe time-bounded and job-bounded abstractions. In Sec. IV, we describe our encoding method. In Sec. V, we extend our model with locks, and in Sec. VI describe partial-order reduction. In Sec. VII, we describe our case study and experimental results. Finally, we discuss related work in Sec. VIII, and conclude in Sec. IX.

II. Preliminaries

A task \( \tau \) is a tuple \((I, T, P, C, A)\), where \( I \) is a task identifier, \( T \) – a bounded procedure (i.e., unbounded loops or recursion) called the task body, \( P \) – a period, \( C \) – the worst case execution time of \( T \), and \( A \), called the release time, is the time at which the task is first enabled. An \( N \)-task periodic program \( C \) is a set \( \{\tau_0, \ldots, \tau_{N-1}\} \) of \( N \) tasks. For simplicity, we assume that the id of task \( \tau_i \) is \( i \).

In this paper, we restrict the priorities of the tasks to be rate-monotonic – tasks with smaller period have higher base priority. For simplicity, we assume that the index of a task represents its base priority. Thus, a task with a lower id has a lower base priority (and higher period).

A periodic program is executed by running each task periodically, starting at the release time. For \( k \geq 0 \) the \( k \)-th job of \( \tau_i \) becomes enabled at time \( A_i^k = A_i + k \times P_i \). The execution is asynchronous and priority-sensitive – at each point the CPU is given to an enabled task with the highest priority. Priorities can change dynamically, but must avoid priority inversion – when a low base priority task preempts a higher base priority task. This is known, somewhat misleadingly, as a fixed-priority preemptive scheduling.

Formally, the semantics of an \( N \)-task periodic program \( C = \{\tau_0, \ldots, \tau_{N-1}\} \) is the asynchronous concurrent program:

\[
\forall i = 0 \ldots N-1 \quad k_i := 0 \quad \textbf{while}(\text{WAIT}(\tau_i, k_i)) \quad (T_i; k_i := k_i + 1) \quad (1)
\]

where \( k_i \) is a numeric variable and \( \text{WAIT}(\tau_i, k_i) \) returns FALSE if the current time is greater than \( A_i^{k_i} \), and otherwise it disables \( \tau_i \) until the time is \( A_i^{k_i} \) and then returns TRUE.

An execution of each task body \( T_i \) in (1) is called a job. A job’s arrival is the time when it becomes enabled (i.e., \( \text{WAIT}(\tau_i, k_i) \) in (1) returns TRUE); start and finish are the times when its first and last instructions are executed, respectively; response time is the difference between its finish and arrival times. The response time of a task is the maximum of response times of all of its jobs in all possible executions.

Note that \( \text{WAIT} \) in (1) returns TRUE if a job has finished before its next period. If this is always the case, i.e., \( \text{WAIT} \) never returns FALSE, then the program is called schedulable.

Formally, a periodic program \( C \) is schedulable if for each task \( \tau_i \), the response time \( RT_i \) is less than the period \( P_i \). Response times are computed using Rate Monotonic Analysis (RMA) [14]. For a periodic program \( C = \{\tau_0, \ldots, \tau_{N-1}\} \), the response time \( RT_i \) of task \( \tau_i \) is the smallest solution to the following equation

\[
RT_i = C_i + \sum_{i<k<N} \left( \frac{RT_k}{P_k} \right) \cdot C_k.
\]

Intuitively, the response time of a task is equal to its worst-case execution time plus the time taken by all higher-priority tasks that preempted it. \( RT_i \) is computed by solving (2) iteratively starting with \( RT_0 = C_0 \) [14]. Note that for the highest-priority task the response time \( RT_{N-1} \) is its execution time \( C_{N-1} \).

Example 1 Consider the task set:

<table>
<thead>
<tr>
<th>Task</th>
<th>( C_i )</th>
<th>( P_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_2 )</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>( \tau_1 )</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>( \tau_0 )</td>
<td>8</td>
<td>16</td>
</tr>
</tbody>
</table>

Solving (2) gives us \( RT_2 = 1 \), \( RT_1 = 3 \) and \( RT_0 = 16 \). A schedule demonstrating these values is shown in Fig. 1.

In this paper, we are interested in logical properties of periodic programs. We assume that any program we analyze meets its basic timing constraints. For that reason, we only deal with schedulable periodic programs.

III. Time-Bounded Periodic Programs

In this section, we present time-bounded semantics of periodic programs and define a job-bounded abstraction that is the formal foundations of our verification technique.
Given a periodic program \( C = \{\tau_0, \ldots, \tau_{N-1}\} \) and a time-bound \( \mathcal{W} \), the time-bound program \( C_{\mathcal{W}} \) executes like \( C \) for time \( \mathcal{W} \) and then terminates. We assume that \( \mathcal{W} \) is divisible by the period of each task. Furthermore, we assume that the first job of each task finishes before its period, i.e., \( A_i \leq P_i - RT_i \).

Under these assumptions, the time bound imposes a natural limit on the number of jobs \( J_i \) of each task:

\[
J_i = \left\lfloor \frac{\mathcal{W}}{P_i} \right\rfloor. \tag{3}
\]

Therefore, the semantics of \( C_{\mathcal{W}} \) is equivalent to the asynchronous concurrent program:

\[
\|_{i=0}^{N-1} k_i := 0; \textbf{while}(k_i < J_i \land \text{WAIT}(\tau_i, k_i)) \{ T_i; k_i := k_i + 1 \}. \tag{4}
\]

This is analogous to the semantics of \( C \) (in (1)) except that each task \( \tau_i \) executes \( J_i \) jobs.

**Job-bounded Abstraction.** Since we are interested in logical properties, we need to abstract the absolute time in (4) with relative order of execution. A simple abstraction is to interpret \( \text{WAIT} \) as a non-deterministic delay, effectively replacing a time-bound with a job-bound. We further refine this abstraction using the following observation about RMA (2). Let \( \tau_i \) and \( \tau_j \) be two tasks of a schedulable periodic program \( C \) such that \( i < j \). Then RMA defines the **preemption bound** of \( \tau_i \) by \( \tau_j \), written \( PB_i^j \), as follows

\[
PB_i^j = \left\lceil \frac{RT_i}{P_j} \right\rceil. \tag{5}
\]

That is, \( PB_i^j \) is an upper bound on the number of times \( \tau_j \) can preempt \( \tau_i \). Thus, in addition to treating \( \text{WAIT} \) as a non-deterministic delay, we only allow for a job of task \( \tau_j \) to preempt a job of task \( \tau_i \) if \( j > i \) and it does not violate the preemption bound \( PB_i^j \). In other words, we only schedule at most \( PB_i^j \) jobs of \( \tau_j \) while a job of \( \tau_i \) is active. We call this the job-bounded abstraction of \( C_{\mathcal{W}} \) and denote it by \( C_{J(\mathcal{W})} \).

**Theorem 1 (Soundness of Job-bounded Abstraction)** For a periodic program \( C = \{\tau_0, \ldots, \tau_{N-1}\} \) and a time-bound \( \mathcal{W} \) s.t. \( \forall i \cdot (A_i \leq P_i - RT_i) \land (\mathcal{W}|P_i) \), every execution of \( C_{\mathcal{W}} \) is also an execution of \( C_{J(\mathcal{W})} \).

Obviously, job-bounded abstraction is incomplete, i.e., \( C_{J(\mathcal{W})} \) may have more executions than \( C_{\mathcal{W}} \). The primary reason being that the preemption bounds are only upper bounds since they are computed from the worst-case execution times, and rounded upwards in (5). The incompleteness due to rounding up is shown by the following example.

**Example 2** Let \( C = \{\tau_0, \tau_1\} \) such that \( P_0 = 25 \), \( RT_0 = 22 \) and \( P_1 = 10 \), \( RT_1 = 10 \), and \( \mathcal{W} = 100 \). Then, \( PB_0^1 = 3 \) and, therefore, in \( C_{J(\mathcal{W})} \) two consecutive jobs of \( \tau_0 \) can be preempted three times, each, by jobs of \( \tau_1 \). However, in \( C_{\mathcal{W}} \) two consecutive jobs of \( \tau_0 \), taken together, can be preempted at most five times by jobs of \( \tau_1 \).

In the next section, we give an algorithm for constructing and verifying the job-bounded abstraction \( C_{J(\mathcal{W})} \) from a periodic program \( C \).

IV. **SEQUENTIALIZATION OF A PERIODIC PROGRAM**

We use a two-step approach to verify an \( N \)-task periodic program \( C = \{\tau_0, \ldots, \tau_{N-1}\} \) under a time bound \( \mathcal{W} \). The first step, sequentialization, outputs a non-deterministic sequential program with assume statements \( S \), as shown in Alg. 1. The program \( S \) is semantics preserving w.r.t. \( C_{J(\mathcal{W})} \) (see Theorem 2). The second step is the verification of \( S \) with an off-the-shelf program verifier. In the rest of this section, we focus on sequentialization.

**A. Sequentialization: Intuition**

Our key insight is that any execution \( \pi \) of \( C \) can be partitioned into scheduling **rounds** in the following way: (a) \( \pi \) begins in round 0, and (b) a round ends and a new one begins every time a job ends (i.e., the last instruction of some task body is executed). For example, the bounded execution shown in Fig. 1 is partitioned into 7 rounds as follows: round 0 is the time interval \([0, 1]\) – the end of the first job of \( \tau_2 \), round 1 is \([1, 3]\) – the end of the first job of \( \tau_1 \), round 2 is \([3, 5]\) – the end of the second job of \( \tau_2 \) (note that there is only one job of \( \tau_0 \) and it ends at time 16), round 3 is \([5, 9]\), etc.

Observe that in each round, the tasks are executed sequentially in the order of their priority starting with the task of the lowest priority. Furthermore, a bounded execution in which exactly \( k \) jobs start and end has exactly \( k \) rounds.

The basic idea of our sequentialization is to reduce a bounded concurrent execution with \( k \) jobs into a sequential execution with \( k \) rounds. Initially, jobs are allocated (or scheduled) to rounds. Then, each round is executed independently (and sequentially) starting from a guessed initial state. Finally, we check for each \( 0 \leq i < (k - 1) \), that the guessed initial state at round \( i + 1 \) is the final state of round \( i \).

Our sequentialization is inspired by the work of Lal and Reps [9], but differs from it in several significant ways. First, we deal with periodic programs and not non-deterministically scheduled concurrent programs. Second, since we are not exploring non-deterministic schedules, we use a more refined scheduling scheme than the non-deterministic round-robin used in [9]. Third, we partition each task (or, correspondingly, a thread in [9]) into jobs and preserve **all** executions in which **all** jobs terminate. In contrast, [9] only preserves executions with a bounded number of thread-preemptions. Finally, we take into account that preemption between tasks must respect priorities and preemption bounds.

**B. Sequentialization: Details**

The sequential program \( S \) starts in MAIN (see Alg. 1). It first allocates jobs to rounds (line 5), initializes global variables (lines 6–8), executes all jobs of all tasks sequentially starting with the first job of the lowest priority task (lines 9–13), and finally checks correctness of guessed variables and assertions (lines 14–15).

**Job scheduling.** A job schedule is a pair of mappings **start** and **end** that map each job of each task to a starting and ending round, respectively. We say that a job schedule is legal iff it satisfies the following three properties: (a) jobs are sequential
– for a given task, for any pair \( i < j \), job \( i \) starts and finishes (i.e., precedes) job \( j \), and (b) jobs are well-nested – if a higher and a lower priority jobs overlap, then the higher priority job must start and end within the rounds of the lower priority one, and (c) jobs respect preemption bounds – no more than \( PB_{t_i}^j \) jobs of task \( t_j \) are scheduled inside any job of task \( t_i \).

**SCHEDULE** fills arrays \( \text{start} \) and \( \text{end} \) with a non-deterministic legal job schedule. Line 24 ensures that the jobs are sequential, while line 25 ensures that they are well-nested, and line 26 ensures that they respect preemption bounds. To understand line 26, suppose that \( PB_{t_3}^3 = 3 \), \( j_2 = 4 \) and that \( j_2 \) is nested in \( j_1 \). Then, to satisfy \( PB_{t_3}^j \), we require that job 1 of \( t_2 \) has ended before \( j_1 \) started. Otherwise, because the jobs are sequential and well-nested, \( j_1 \) contains jobs 1, 2, 3, and 4 of \( t_2 \) – that is, it is preempted by more than three \( t_2 \) jobs.

**Global variables.** For every global variable \( g \), we create two arrays \( g[i] \) and \( v_g[i] \) such that \( g[i] \) is the value of \( g \) in round \( i \), and \( v_g[i] \) is the guess of the initial value of \( g \) in round \( i \). The element \( g[0] \) is initialized in line 7 to the user-specified initial value \( v_g \), and each other elements \( g[r] \) is assigned the corresponding initial guess \( v_g[r] \) in line 8.

In addition, a variable \( rnd \) tracks the current round, \( job \) tracks the current job, and \( endRnd \) is the scheduled end round of the current job.

**Tasks.** For each task \( t \), **MAIN** uses a modified version \( T_t \) obtained from the original task body \( T_t \) by preceding each statement \( st \) with a call to **CS** to emulate a preemption, and replacing every global variable \( g \) in \( st \) with \( g[rnd] \). This is based on an assumption, without loss of generality, that each global variable \( g \) is explicitly loaded and stored, i.e., \( g \) only appears in statements of the form \( g := l \) or \( l := g \), where \( l \) is a local variable.

**Preemption.** **CS** models a preemption (a context switch to a higher priority task) by increasing non-deterministically the value of \( rnd \) to the round in which this task resumes execution. However, not every round between the start and end of the current job is legitimate. Line 21 ensures that the execution is not resumed in a round used by a higher-priority task. **CS** returns \( \text{TRUE} \) iff a preemption has occurred. This value is used later in Sec. VI.

**Prophecy-Check.** Line 14 ensures that the value of each global variable at the end of a round is equal to its guessed value at the beginning of the next round.

**Assertions.** Assertions need special handling. They can only be checked after the guesses have been validated via Prophecy-Check. Without loss of generality, we assume a single call to assert the body of each task. We use an array \( \text{localAssert} \) that maps a task and a job to the value of the assertion in it. The element \( \text{localAssert}[t][j] \) is initialized to \( \text{TRUE} \) (line 6), set to the value of the asserted expression (line 29), and asserted to be \( \text{TRUE} \) (line 15) after the Prophecy-Check.

Our sequentialization procedure is semantic preserving as expressed in the following theorem.

**Theorem 2** Let \( C \) be a periodic program and \( W \) a time-bound satisfying the conditions in Theorem 1. Then, for every execution \( \pi \) of \( C_{J(W)} \) that violates an assertion in job \( j \) of task \( t \), there is a corresponding execution \( \pi' \) of the sequential program \( S \) that violates \( \text{localAssert}[t][j] \), and vice versa.

**V. LOCKS**

We support two types of locks: preemption locks and priority ceiling locks. These locks are common in periodic programs because they are non-blocking (acquiring a lock always succeeds) and avoid common pitfalls such as priority inversion and deadlocks.

A periodic program has a single preemption lock \( pl \). Acquiring \( pl \) disables the scheduler, preventing all priority-based preemptions. Releasing \( pl \) re-enables the scheduler. An example of such a mechanism is the \text{taskLock / taskUnlock} routines in VxWorks [15].

Priority ceiling locks, or priority locks for short, are based on dynamically raising the priority of the current job. Each priority lock \( lck \) is associated with a fixed priority \( \text{LOCK_PRIORITY}(lck) \), which is given to a task if it acquires \( lck \). It is illegal for a task with current priority \( p \) to acquire a priority lock \( lck \) such that \( \text{LOCK_PRIORITY}(lck) \) is less than \( p \). Releasing a lock restores the priority. Priority locks are used in the Highest Locker-Priority (HLP) protocol [16], where the priority of a resource \( r \) is as high as the priority of any task accessing \( r \). This guarantees mutual exclusion (assuming there is only one CPU) while avoiding blocking, deadlocks, and priority inversion. Multiple priority locks must be acquired in increasing order of priority, but can be released in an arbitrary order. We now show our encoding of these locks.

**Preemption locks.** The preemption lock \( pl \) is modeled by introducing a Boolean variable \( lock \) into \( T_t \). Specifically, \( lock = \text{TRUE} \) iff the current task has acquired \( pl \). The variable is \( \text{FALSE} \) initially and is reset to \( \text{FALSE} \) at the end of a job. Finally, calls to **CS** are conditioned by \( \text{lock} = \text{FALSE} \).

**Priority locks.** To model priority locks, we need to model the dynamic priority of each task. Let \( \text{BASE_PRIORITY}(t) \) be a function that returns the base priority of task \( t \). We introduce an array \( \text{priority} \) such that for every round \( r \), \( priority[r] \) is the priority of the task currently executing in round \( r \). The array is maintained by the function \( \text{WRAPPER} \) (shown in Alg. 2) that wraps the body \( T_t \) of each task \( t \). The wrapper function saves the current priority (line 2), ensures that it is below the base priority of the current task (line 3), raises the priority to the priority of the current task (line 4), executes the task body (line 5), and finally resets the priority (line 6). Note that the task body is only executed if the task has higher priority than the current dynamic one, and that the priority can be raised further inside the task body itself.

We model priority locks with two functions \( \text{GETLOCK} \) and \( \text{RELEASELOCK} \) shown in Alg. 2. The set of all locks held by a task is maintained in a set \( \text{lockSet} \) that is local to each task. Information about locks of other tasks is propagated through priorities. Acquiring a lock (\( \text{GETLOCK} \)) raises the dynamic priority to the one of the lock, releasing a lock (\( \text{RELEASELOCK} \)) resets the priority to the base priority of the task or to the priority of the highest lock in the \( \text{lockSet} \) in
case this set is not empty. Note that this allows for acquiring multiple locks and releasing them in an arbitrary order. We check that the priority of the locks is assigned correctly (i.e., acquiring a lock must never lower the dynamic priority) by adding an assertion that checks this in line 8 of GETLOCK.

Correctness. We need to show that CS and SCHEDULEJOBS in Alg. 1, which are based on the base priorities, are still correct when priorities can change due to priority locks.

First, we need to show that the schedules permitted by SCHEDULEJOBS cover the legitimate schedules in the presence of priority locks. This is indeed the case because priority-locks cannot lead to priority inversion – a situation in which a job with a low base priority preempts a job with a high base priority. We will demonstrate this using jobs j1 and j2 with base priorities 1 and 2, respectively. j1 cannot preempt j2 even if it raises its own priority to 3, simply because j2 is not running when j1 acquires the lock. It can, therefore, only delay the start time of j2, not preempt it. Thus, it is not necessary to explore schedules in which a low-base-priority job preempts a high-base-priority job.

Next, consider CS. Line 21 in CS guarantees that j1 does not resume control in a round in which j2 is still active. If j1 raises its priority then j2 cannot preempt it, and therefore this constraint blocks a computations in which j1 resumes when j2 was initially scheduled to run. Such a computation is illegal, however, because it corresponds to a preemption of a high-priority job j1 by a lower-priority job j2, it is accordingly blocked by the assume statement on line 3 of Tt-WRAPPER.

Finally, since j1 can raise its priority, it seems that we also need the opposite constraint (i.e., that j2 does not resume control in a round in which j1 is still active). However, there is no need for this constraint because SCHEDULEJOBS does not allow a schedule in which j1 preempts j2.

VI. PARTIAL-ORDER REDUCTION

Computations of a concurrent system have a natural partitioning: two computations are in the same class iff they reach the same observable states. Thus, for verification, it suffices to examine only one representative from each class. This is known as Partial-Order Reduction (POR) [17], [18].

POR is used widely and effectively in explicit-state Model Checking (e.g., [19]). Recently, it has been shown to be
Algorithm 2 Priority locks.

1: function $T_i$-WRAPPER( )
2: $sp := priority[rnd]$
3: assume($sp \leq BASEPRIORITY(t))$
4: priority[rnd] := BASEPRIORITY(t)
5: $T_i( )$
6: priority[rnd] := $sp$
7: function GETLOCK(Lock $lck$)
8: assert(LockPRIORITY($lck$) \geq priority[rnd])
9: lockSet := lockSet ∪ $\{lck\}$
10: priority[rnd] := LockPRIORITY($lck$)
11: function RELEASELOCK(Task $t$, Lock $lck$)
12: lockSet := lockSet \ $\{lck\}$
13: if lockSet = 0 then
14: priority[rnd] := BASEPRIORITY(t)
15: else
16: priority[rnd] := priority of highest lock in lockSet

Algorithm 3 A fragment of $T_i$ from Alg. 1 with POR. Only the case of a store to shared variable $g$ is shown.

1: function $T_i( )$
2: Same as $T_i$, but
3: each statement ‘$g := l$’ is replaced with:
4: oldRnd := rnd
5: if CS($t$) then
6: $W_g[oldRnd] := R_g[oldRnd] := FALSE$
7: assume($W_g[rnd] \lor R_g[rnd]$)
8: $W_g[rnd] := TRUE$
9: $g[rnd] := l$

Fig. 2. An encoding for Example 3.

Example 3 Under semantic POR, the two assignments

1: $x := g ; g := y$

in $T_i$, where $x, y$ are local and $g$ is shared, become the sequence in $T_i$ that appears in Fig. 2.

1: oldRnd := rnd
2: if CS($t$) then
3: $W_g[oldRnd] := R_g[oldRnd] := FALSE$
4: assume($W_g[rnd]$)
5: $R_g[rnd] := TRUE$
6: $x := g[rnd]$
7: oldRnd := rnd
8: if CS($t$) then
9: $W_g[oldRnd] := R_g[oldRnd] := FALSE$
10: assume($W_g[rnd] \lor R_g[rnd]$)
11: $W_g[rnd] := TRUE$
12: $g[rnd] := y$

Let $S^{\text{po}}$ denote the sequentialization with POR. The following theorem shows that it is semantics preserving.

Theorem 3 Let $C$, $W$ and $S$ be as in Theorem 2 and $S^{\text{po}}$ the corresponding POR. Then, for every execution $\pi$ of $S$ that violates a local assertion $\text{localAssert}(t)[j]$ of task $t$ and job $j$ there is a corresponding execution $\pi'$ of $S^{\text{po}}$ that violates $\text{localAssert}(t')[j]$, and vice versa.

VIII. CASE STUDIES

We implemented our approach in a tool called REK. REK is built on top of CIL [21]. It takes as input C programs annotated with entry points of each task, their periods, worst case execution times, and the time bound $W$. The output is a sequential C program $S$ that is then verified by CBMC [22].
To evaluate our approach, we have used REk to verify several periodic programs. In the rest of this section, we report on this experience. The tool and the case studies are available at: http://www.andrew.cmu.edu/user/arieg/Rek.

**Robot controller.** The NXTway-GS controller, nxt for short, runs on nxtOSEK [2] – a real-time operating system ported to the LEGO MINDSTORM platform. nxtOSEK supports programs written in C with periodic tasks and priority ceiling locks. It is the target for Embedded Coder Robot NXT – a Model-Based Design environment for using Simulink models with LEGO robots.

The basic version of the controller has 3 periodic tasks: a balancer, with period of 4ms, that keeps the robot upright and monitors the bluetooth link for user commands, an obstacle, with period 50ms, that monitors a sonar sensor for obstacles, and a 100ms background task that prints debug information on an LCD screen.

We verified several versions of this controller. All of the properties verified involved the high-frequency balancer task. The balancer goes through 3 modes of execution: INIT, CALIBRATE, and CONTROL. In INIT mode all variables are initialized, and in CALIBRATE a gyroscope is calibrated. After that, balancer goes to CONTROL mode in which it iteratively reads the bluetooth link, reads the gyroscope, and sends commands to the two motors on the robot’s wheels.

The results are shown in the top part of Table I. We have used \( W = 100 \text{ms} \), which is the minimum time needed for all tasks to execute at least once. We did not enable semantic POR since it was irrelevant in this case (all shared variables were accessed by all tasks in all paths).

Experiments nxt.ok1 (nxt.bug1) check that the balancer is in a correct (respectively, incorrect) mode at the end of the time bound. Experiment nxt.ok2 checks that the balancer is always in one of its defined modes. Experiment nxt.bug3 checks that whenever balancer detects an obstacle, the balancer responds by moving the robot. We found that since the shared variables are not protected by a lock there is a race condition that causes the balancer to miss a change in the state of obstacle for one period. Experiment nxt.ok3 is the version of the controller where the race condition has been resolved using locks.

For the second part of the robot case study, we modified the original design to separate handling of each sensor by a separate task. Our design, called ‘aso’ has 3 tasks: balancer, observer, and bluetooth. The first two are the same as before, and the bluetooth is responsible for the bluetooth communication. We wanted a design in which the balancer task is lock-free, which was challenging. During our design, we unintentionally introduced subtle concurrency errors which were detected by REk.

The results of these experiments are shown in the second part of Table I. We checked consistency of communication between the tasks. The experiments are: aso.bug1 and aso.bug2 – initial versions with inadequate locking leading to race conditions. aso.ok1 is a correct design with preemption locks. aso.bug3 is our first attempt at a lock-free implementation that was fundamentally flawed and had to be abandoned. aso.bug4 and aso.ok2 are a buggy and a correct version of the final design in which obstacle and bluetooth synchronize via priority locks and balancer is lock-free.

During the case study, we found it very convenient to increase the period of the highest priority task (thus decreasing its frequency). In many cases, this dramatically reduced verification time, while allowing us to draw meaningful conclusions from the counterexamples. Of course, this approach is not sound in general. Fig. 3 shows the relationship between the analysis time and the frequency of balancer for a correct and an incorrect version of the controller. In case the design is buggy, the time increased monotonically with the frequency. However, for a safe design, the time behaves erratically: e.g., increasing the frequency from 20 to 26 made it easier to verify. Such erratic behavior is common with SAT.

**Reader-Writer.** Reader-Writer (RW) is a common communication pattern in concurrent programs. We implemented three

### Table I

<table>
<thead>
<tr>
<th>Name</th>
<th>Program Size</th>
<th>SAT Size</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OL</td>
<td>SL</td>
<td>GL</td>
</tr>
<tr>
<td>nxt.ok1</td>
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</tr>
<tr>
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lock-free flavors of a RW protocol (RW1, RW3, and RW5), and their counterparts with locks (RW2, RW4, and RW6). We checked consistency of communication between the tasks. Each protocol was analyzed with 3 to 6 tasks (depending on the protocol), with \( V \) such that every task executes once, and with an increasing number of shared variables. The results are shown in Table I. For each protocol, we only report on the hardest instance solved in under 10 mins. In these examples, semantic POR yields significant reduction in verification time. The results with POR are shown in Table I in rows named “PO”. Note that in all cases, the number of variables and clauses with POR is larger, yet the verification time is smaller.

VIII. RELATED WORK

There is a large body of work in verification of logical properties of both sequential and concurrent programs (see [23] for a recent survey). However, these techniques abstract away time completely, by assuming a non-deterministic scheduler model. In contrast, we use a priority-sensitive scheduler model, and abstract time partially via our job-bounded abstraction.

In both cases, ours and LR, the number of variables is proportional to the number of rounds, \( R \). However, LR allows more computations since it does not enforce priority constraints. In addition to yielding fewer false warnings, our approach guarantees better coverage for the same number of variables, as shown below:

Take two programs: (i) an \( N \)-task periodic program \( C \) with a time bound that permits exactly one job per task; (ii) the analogue of \( C \), called \( C' \), in a non-real-time setting, i.e., \( N \) threads scheduled non-deterministically, each executing one task of \( C \). Consider the value of \( R \) required to cover all reachable states, in our encoding \( S \) vs. the LR encoding of \( C' \). For LR, \( R \) is the number of possible context switches, which by itself is proportional to the number of statements over shared variables in \( C' \). This value of \( R \) is needed to explore a pathological path in \( C \) where, in each round, a single thread is executed and the threads are picked in reverse-round-robin order. In contrast, our approach only needs \( R = N \), a much smaller value in most practical cases. In fact, the pathological path above is illegal, since scheduling tasks in reverse-round-robin order violates priority constraints.

IX. CONCLUSION

Periodic programs, i.e., periodic RTES with rate-monotonic scheduling, are an important sub-class of embedded real-time software. In this paper, we address the time-bounded verification of safety properties of periodic program implementations. We present a solution involving two steps – convert the target periodic program to a non-deterministic sequential program, and then verify it with an off-the-shelf verification tool. Our approach is sound, preserves both data- and control-flow, and abstracts the effect of time via preemption-bounds. Some of our techniques are applicable to other types of systems. Specifically, note that we only used the assumption that the verified system follows RMS in order to compute preemption bounds. Other periodic RTES can be modeled with our method by either supplying these bounds as part of the input, or by removing line 26 in Alg. 1 (this may hinder completeness, however). In addition, our partial order reduction is not restricted to periodic RTES, and is applicable when analyzing general multi-threaded programs.

We have implemented our approach in a tool, and used it to identify subtle concurrency errors in a robot controller. We believe that our work opens up several avenues for future work in real-time software verification, notably unbounded verification of periodic programs and the use of automated abstraction refinement techniques.

REFERENCES


