Solution-Based Congestion Games

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Abstract

In this paper we develop the theory of potential of cooperative games for semivalues, characterize congestion models that are defined by semivalues, and suggest an application of these results to combinatorial auctions, which may explain the success of the Iowa electronic market.

1 Introduction

Models of congestion come with many real-life stories and in various mathematical forms. They seem to originate at transportation engineering [37],[1]

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[1]In today's game theoretic language,the congestion models that were discussed by [37] define non-atomic games in strategic form [30], where the non-atomic part refers to the

and they have been analyzed by several researchers from various additional fields, in particular computer science[2], communication networks[3], and economics/game theory.[4]

Our starting point is the model of [27].

A congestion form is defined by a finite set of players, each of which holding one unit of goods, a finite set of facilities, and per-unit payoff functions associated with the facilities. Each player must use a subset of facilities in order to make its unit of goods valuable. The non-empty set of feasible subsets of facilities is player-specific. When a player chooses a subset of facilities her per-facility payoff depends on the number of other players that decide to use the facility, and her total payoff is the sum of payoffs of the facilities in this subset.

Each congestion form F defines a game in strategic form, Γ_F, which is called a congestion game. In Γ_F the strategy set of a player is her set of feasible subsets of facilities, and her payoff function is described above. The distinction between congestion forms and congestion games is important. The form, which is also a sort of game, contains more information than its associated game in strategic form.

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number of players. Such models are not discussed in this paper, which focuses on discrete models. In the context of networks, equilibrium in such nonatomic models is sometimes referred to as Wardrop Equilibrium. For some research concerning such non-atomic games see, e.g., [29].

[2]Most research in computer science has been done in the context of the price of anarchy. The price of anarchy of a game with positive payoffs is the ratio of the maximal expected social payoff (sum of payoffs) to the maximal social payoff obtained at a mixed-strategy equilibrium. The concept was initiated in [15, 26], and has been extensively analyzed for various congestion games . See, e.g., [28].

[3]See [1] for references and for a salute-worthy attempt to unify the research.

[4]See e.g., [27, 21, 18, 10, 29, 34, 16].

In [19] a natural generalization of congestion forms (games) is discussed. in these forms the facility payoff functions are player-specific. The paper [19] deals mainly with the existence and properties of representations of games in strategic form by congestion models. A previous representative existence theorem is the one of [21] stating that every potential game is isomorphic to a congestion game.[5]

In particular it is proved in [19] the somewhat surprising result that every game in strategic form is isomorphic to a PS-congestion game.

In some contexts it is useful to consider congestion forms, which generalize PS-congestion forms. In this generalized model the facility payoff functions are not only player-specific, but also depend on the identity of the users of the facility. We call such forms ID-congestion forms, and their associated games ID-congestion games. Obviously every ID-congestion game is a PS-congestion game. Therefore the fact that every game in strategic form is isomorphic to a PS-congestion game seems to indicate that there is no advantage in further generalizing PS-congestion forms to ID-congestion forms. However, and unfortunately, it was shown in [19] that any "reasonable" isomorphism operator that transforms games in strategic form to PS-congestion games is not computationally efficient. In Section 3 we focus on ID-congestion games that are defined by a solution operator for cooperative games. Roughly speaking, a solution-based form is defined by a congestion model in which every facility is associated with a cooperative game, and the payoff for an agent who chooses this facility is his index or value in this game restricted to

[5]Some congestion models are represented by graphs. Graphical representations of games have been analyzed in the literature of computers science and artificial intelligence. In some of these representations the focus is on dependencies among players' utility functions (see, e.g., [13, 14, 22, 35]). Other types of representations focus on actions' dependencies– see [17, 24].

the set of all users of this facility. A very simple version of a game derived from such a model was analyzed in [21], where it was used to characterize the Shapley value. Another, elegant, characterization of potential games by the Shapley value was given in [34]. In order to analyze solution-based forms and games we are required to generalize the potential theory of cooperative games [8]. This is done in Section 2. Section 4 suggests an application of our results via the concept of combinatorial auctions [4] and Information Markets.[6] This particular section does not present concrete mathematical results, but rather an initial idea.

2 Semivalues

Let N be a finite set with n elements, $n \geq 1$. Elements of N are called *players*. Any subset of N is called a *coalition*. We denote by $C(N)$ the set of all coalitions. For a coalition S, we denote $N \setminus S$ by S^c. For a player i and a coalition S we will write $S \cup i$ for $S \cup \{i\}$, and $S \setminus i$ for $S \setminus \{i\}$.

A *transferable utility cooperative game* on N (in short: a *cooperative game*) is a set function $v : C \rightarrow \Re$, where $\Re$ denotes the set of real numbers, with $v(\emptyset) = 0$. $v(S)$ is interpreted as the total joint revenue for the members in S if the coalition S is formed, and it is referred to as the *worth* of S. For every game v and for every coalition S we denote by $v_S \in G$ the game defined by $v_S(T) = v(T \cap S)$ for every $T \in C(N)$. v_S is called a *subgame* of v. The set of all cooperative games with the set of players N is a linear space denoted by $G(N)$. Let $v \in G(N)$. We denote by $G_v(N)$ the set of all subgames of v. That is, $G_v(N) = \{v_S | S \in C(N)\}$. When the set of players N is clear we will write G, C, G_v for $G(N), C(N), G_v(N)$, respectively.

[6]See e.g., http:\\ www.biz.uiowa.edu \ iem\

A *solution* on G is a function $\tau : G \to \Re^N$. $\tau v(i)$ is interpreted as the share of i or as the index of power of i.

The most well-known solution is the Shapley value [31]. The Shapley value has two main variations: quasivalues and semivalues.[7] In this paper we focus on semivalues.

By [5] every semivalue on $G(N)$ is uniquely defined by a vector $\beta = (\beta_0, \cdots, \beta_{n-1})$ of non negative numbers such that $\sum_{s=0}^{n-1} \binom{n-1}{s} \beta_i(s) = 1$. For every such β, and for every $i \in N$ we denote by P^i_β the probability distribution on $C(N \setminus i)$ defined as follows:

$$P^i_\beta(S) = \beta_{|S|} \quad \text{for every } S \subseteq N \setminus i,$$

where $|S|$ denotes the number of players in S. The semivalue associated with β, ψ_β is defined as follows:

$$\psi_\beta v(i) = \sum_{S \in C(N \setminus i)} (v(S \cup i) - v(S)) P^i_\beta(S), \quad \text{for every } i \in N.$$

The Shapley value is the semivalue defined by the vector β for which $\beta_j = \frac{1}{n\binom{n-1}{j}}$ for every $0 \le j \le n-1$. For every solution τ we define $I_\tau : G \to R$ by $I_\tau(v) = \tau v(N)$, where for $x \in R^N$, and a coalition S, $x(S) = \sum_{i \in S} x_i$; $\sum_{i \in \emptyset} x_i$ is defined to be 0. $\tau v(N)$ is called the *total index of* v *with respect to* τ. Let τ be a solution and let ψ be a semivalue. We say that τ is ψ-efficient if $I_\tau = I_\psi$. It was proved in [20] that if the total index operators of two semivalues coincide then the two semivalues coincide. That is $\psi_1 = \psi_2$ if $I_{\psi_1} = I_{\psi_2}$. In particular, it follows that if ψ_1 is ψ_2-efficient then $\psi_1 = \psi_2$.

Remark It is desirable to axiomatically characterize total index operators, but we could not find a simple characterization.[8]

[7] Semivalues were introduced by [5]. For a survey of quasivalues and semivalues see [20].

[8] Note that for the Shapley value φ, $I_\varphi v = v(N)$ for every $v \in G$.

2.1 Semivalues and potentials

Potential functions that characterize the Shapley value were defined and analyzed in [8]. Let ψ be a semivalue. Let $H \subseteq G$ be subgame closed. That is $v_S \in H$ for every $v \in H$ and for every $S \subseteq N$.[9] A *ψ-potential* on H is a function $Q : H \to \Re$ that satisfies $Q(0) = 0$, and

$$\sum_{i \in S} (Q(v_S) - Q(v_{S \setminus i})) = I_\psi(v_S) \quad \text{for every } v \in H, \text{ and every } \emptyset \neq S \subseteq N.$$

Theorem 1 *Let H be a subgame closed subset of G, and let ψ be a semivalue. There exists a unique ψ-potential on H, Q. Moreover,for every $i \in N$*

$$\psi v_S(i) = Q(v_S) - Q(v_{S \setminus i}) \quad \textit{for every } v \in H, \textit{ and for every } \emptyset \neq S \subseteq N. \tag{1}$$

Proof: When ψ is the Shapley value the proof is given in [8]. It is easy to show that the method of proof works also for semivalues as long as we show that every semivalue satisfies the balanced contribution property ([23]):[10]

$$\psi v(i) - \psi v_{N \setminus j}(i) = \psi v(j) - \psi v_{N \setminus i}(j) \quad \text{for every } v \in G(N), i, j \in N. \tag{1}$$

For every $0 \leq k \leq n-1$ let ψ_k be the semivalue ψ_{β^k}, where $\beta^k(s) = 0$ for $s \neq k$, and for $s = k$ $\beta^k(k) = \frac{1}{\binom{n-1}{k}}$. It is easy to see that

$$\psi_{\beta^k} v(i) = \beta^k(k) \sum_{S \subseteq N \setminus i, |S| = k} (V(S \cup i) - v(S)) \quad S \subseteq N, i \in N.$$

A simple manipulation reveals that (1) holds for ψ_k for every k. As ψ is a convex combination of ψ_k, $0 \leq k \leq n-1$, (1) holds for ψ. ∎

[9] The Shapley value on certain subgame closed sets was characterized in [25]. Quasi-values on such sets were characterized in [6].

[10] This is called the preservation of difference property in [8].

3 Semivalue-Based Congestion Forms and Games.

Subsections 3.1-3.4 are taken from [19]

3.1 Isomorphic games

A *game in strategic form* is a tuple $\Gamma = (N, (X_i)_{i\in N}, (u_i)_{i\in N})$, where N is a finite set of players, which whenever convenient we take to be $\{1, \cdots, n\}$, X_i is a set of strategies for i, and $u_i : X \to R$ is the payoff function of i, where $X = \times_{i\in N} X_i$. Γ is called a *finite game* if the sets of strategies are finite sets. We say that two games Γ^1 and Γ^2 are *isomorphic* if each of them is obtained from the other by changing the names of the players and the names of the strategies. That is, there exist bijections (i.e., functions which are both one-to-one and onto) $\tau : N^1 \to N^2$ and $\alpha_i : X_i^1 \to X_{\tau(i)}^2$, $i \in N$, such that:

for every $i \in N^1$ and for every $(x_j^1)_{j\in N^1} \in X^1$,

$$u_{\tau(i)}^2((\alpha_j(x_j^1))_{j\in N^1}) = u_i^1((x_j^1)_{j\in N^1}).$$

3.2 Potential games

Let $\Gamma = (N, (X_i)_{i\in N}, (u_i)_{i\in N})$ be a game in strategic form. Let X_{-i} denotes the set of strategy profiles of all players but i. A function $P : X \to R$ is a *potential function* for i if for every $x_i, y_i \in X_i$, and for every $x_{-i} \in X_{-i}$,

$$u_i(x_i, x_{-i}) - u_i(y_i, x_{-i}) = P(x_i, x_{-i}) - P(y_i, x_{-i}).$$

Following [21], Γ is a *potential game*[11] if there exists a function P, which is a potential for every player i.

[11]"Potential game" were defined in [21]. Potential games in the differentiable setup and discrete 2-person potential games were previously discussed in [32, 33], where they

3.3 Congestion forms and congestion games

For the basic model of congestion forms and congestion games we follows [27] and [21].

A *Congestion Form* is a tuple $F = (M, N, (\Sigma_i)_{i\in N}, (w^a)_{a\in M})$, where M is a finite set containing m elements, which are called *facilities*, N is a finite set containing n elements, which are called *players*; For every $i \in N$, $\Sigma_i \subseteq 2^M \setminus \{\emptyset\}$ is a non empty set of subsets of facilities, which is called the *feasible* set of i, and for every $a \in M$, $w^a : [0, \infty) \to \Re$ is the per-unit facility payoff function associated with $a \in M$; If k of the users choose a, each of them receives $w^a(k)$.

Every congestion form $F = (M, N, (\Sigma_i)_{i\in N}, (w^a)_{a\in M})$ defines a game in strategic form Γ_F, in which the set of players is N, Σ_i is the set of strategies of i, and for every $i \in N$ the payoff function of player i is defined on $\Sigma = \times_{i\in N}\Sigma_i$ as follows:

$$u_i(A) = u_i(A_1, \cdots, A_n) = \sum_{a\in A_i} w^a(n^a(A)),$$

where $n^a(A) = |\{j \in N : a \in A_j\}|$.[12] A game Γ in strategic form is called a *congestion game* if $\Gamma = \Gamma_F$ for some congestion form F. Two congestion forms are *equivalent* if they generate isomorphic congestion games. [27] proved that every congestion game is a potential game. [21] proved that every potential game is isomorphic to a congestion game.

are called centralizable games. However, potential functions for various types of games have been used in the literature in several research fields much earlier. See e.g., [37, 2]. Additional references can be found in [21]. Non-atomic potential games were defined in [29].

[12]Hence, only the values of w^a on the set of integers $\{1, \cdots, n\}$ are relevant. However, it is convenient, and it does not restrict the generality, to define w^a on the whole interval $[0, \infty)$.

3.4 Player-specific congestion forms and Player-specific congestion games

When the payoff functions associated with the facilities are player-specific, we get a *congestion form with player-specific facility payoff functions* or, in short a *PS-congestion form.* Formally: A *PS-Congestion Form* is a tuple $F = (M, N, (\Sigma_i)_{i \in N}, ((w_i^a)_{a \in M})_{i \in N})$ such that all components except for the payoff functions are defined as in a congestion form, and $w_i^a : [0, \infty) \rightarrow \Re$ is the i-per-unit facility payoff function associated with $a \in M$; If k of the users choose a, agent i receives $w_i^a(k)$.

Every PS-congestion form $F = (M, N, (\Sigma_i)_{i \in N}, ((w_i^a)_{a \in M})_{i \in N})$ uniquely defines a game in strategic form Γ_F, in which the set of players is N, Σ_i is the set of strategies of i, and for every $i \in N$ the payoff function of player i is defined as follows:

$$u_i(A) = u_i(A_1, \cdots, A_n) = \sum_{a \in A_i} w_i^a(n_a(A)).$$

A game Γ in strategic form is called a *PS-congestion game* if $\Gamma = \Gamma_F$ for some PS-congestion form F. Thus, every congestion form is a PS-congestion form, and every congestion game is a PS-congestion game.

It was proved in [19] that every finite game in strategic form is isomorphic to a PS-congestion game.

This result seems to indicate that there is no advantage in further generalizing PS-congestion forms to congestion models in which for every player i, and for every facility a, i's a-payoff function depends on the identity of the users of a, and not only on their number. However, it was shown in [19]that any "reasonable" isomorphism operator that transforms games in strategic form to PS-congestion games is not computationally efficient. In the next section we deal with a particular generalization of PS-congestion forms and games.

3.5 ID-congestion forms and ID-congestion games

When the payoff functions associated with the facilities are player-specific, and each such player-specific payoff function depends on the identity of the other users and not only on their number, we get an *congestion form with identified users* or, in short an *ID-congestion form.* When defining ID-congestion forms we slightly modify the previous congestion models and we allow non-participation. That is, we allow $\emptyset \in \Sigma_i$. This modification is conceptually natural, and it does not change any of our previous results in this paper as well as Theorem 2 bellow, and the results in [21, 19]. However, we do not know how to prove Theorem 3 bellow without this modification.

Formally: An *ID-Congestion Form* is a tuple $F = (M, N, (\Sigma_i)_{i\in N}, ((w_i^a)_{a\in M})_{i\in N})$ such that N is a set of players, M, is a set of facilities, $\Sigma_i \subseteq 2^M$ is a nonempty set of feasible subsets of facilities, and $w_i^a : \{T \cup i | T \in 2^{N\setminus i}\} \rightarrow \Re$ is the i-per-unit facility payoff function associated with $a \in M$; If S is the set of users that choose a, $i \in S \subseteq N$, agent i receives $w_i^a(S)$. If i chooses the empty set he receives 0 independently of the other players' choices. Every ID-congestion form $F = (M, N, (\Sigma_i)_{i\in N}, ((w_i^a)_{a\in M})_{i\in N})$ uniquely defines a game in strategic form Γ_F, in which the set of players is N, Σ_i is the set of strategies of i, and for every $i \in N$ the payoff function of player i is defined as follows:

$$u_i(A) = u_i(A_1, \cdots, A_n) = \sum_{a\in A_i} w_i^a(S^a(A)), \quad A_i \neq \emptyset$$

where $S^a(A) = \{j \in N | a \in A_j\}$, and $u_i(\emptyset, A_{-i}) = 0$ for every $A_{-i} \in \Sigma_{-i}$.

Thus, every congestion form is a PS-congestion form, and every PS-congestion form is an ID-congestion form. As we noted, every game can be represented with a PS-congestion form, and in particular it can be represented with an ID-congestion form. However we are interested in particular type of ID-congestion forms.

3.6 Solution-based facility payoff functions

Let N be a finite set of players, and let τ be a solution on $G(N)$.

A *τ-based form* is a tuple $G = (M, N, (\Sigma_i)_{i\in N}, (v^a)_{a\in M}, \tau)$ where $M, N, (\Sigma_i)_{i\in N}$ are as in the definition of an ID-congestion form, v^a is a cooperative game in $G(N)$ for every $a \in M$, and τ is a solution on $G(N)$. Every τ-based form, G defines an ID-congestion form $F_G = (M, N, (\Sigma_i)_{i\in N}, ((w_i^a)_{a\in M})_{i\in N})$ in which the facility payoff functions are defined as follows:

$$w_i^a(S) = \tau v_S^a(i), \quad a \in M,\, i \in S \subseteq N.$$

A game in strategic form is τ-based if there exists a τ-form, G such that $\Gamma = \Gamma_{F_G}$.

Theorem 2 *Let Γ be a semivalue-based game in strategic form then Γ is a potential game.*

Proof: Assume Γ is ψ-based, where ψ is a semivalue. Let Q be the unique potential of ψ described in Theorem 1. For every $A \in \Sigma$ define

$$P(A) = \sum_{a\in\cup_{i=1}^n A_j} Q(v^a_{S^a(A)}),$$

where sum over the empty set is defined to equals zero. We show that P is a potential for Γ. Let $i \in N$, let $A_i, B_i \in \Sigma_i$, and let $A_{-i} \in \Sigma_{-i}$. Denote $S^a = \{j \in N \setminus i | a \in A_j\}$. Obviously,

$$P(A_i, A_{-i}) - P(B_i, B_{-i}) = \sum_{a\in A_i\setminus B_i} (Q(v^a_{S^a\cup i}) - Q(v^a_{S^a})) - \sum_{a\in B_i\setminus A_i} (Q(v^a_{S^a\cup i}) - Q(v^a_{S^a})).$$

By (1) ,

$$P(A_i, A_{-i}) - P(B_i, B_{-i}) = \sum_{a \in A_i \setminus B_i} \psi v^a_{S^a \cup i}(i) - \sum_{a \in B_i \setminus A_i} \psi v^a_{S^a \cup i}(i)$$

$$= u_i(A_i, A_{-i}) - u_i(B_i, B_{-i}). \quad \blacksquare$$

Since the Shapley value is a semivalue we conclude:

Corollary 1 *Let Γ be a semivalue-based game in strategic form, which is based on the Shapley value then Γ is a potential game.*

The following is a converse of Theorem 2:

Theorem 3 *Let N be a finite set of players, let M be a finite set of facilities, and let $\Sigma_i \subseteq 2^M$ for every $i \in N$, with $\Sigma_i \neq \emptyset$. Assume in additions that $\{a\} \in \Sigma_i$ for every $i \in N$ and for every $a \in M$.*

Let τ be a solution on $G(N)$ which is ψ-efficient for some semivalue ψ. Let $G = F(N, M, (\Sigma_i)_{i \in N}, \tau, (v^a)_{a \in M})$ be a τ-form. If Γ_{F_G} is a potential game, then the restriction of τ to $H = \cup_{a \in M} G_{v^a}$ coincides with the restriction of ψ to H. Consequently, if $\Gamma_{F_{G(N,M,(\Sigma_i)_{i \in N}, \tau, (v^a)_{a \in M})}}$ is a potential game for every choice of cooperative games $v^a \in G(N)$, $a \in M$, then $\tau = \psi$.

Proof: Let P be a potential for $\gamma = \Gamma_{F_G}$. Without lose of generality we can normalize P to satisfy $P(\emptyset, \emptyset, \cdots, \emptyset) = 0$. Let $a \in M$, and let $S \subseteq N$. We define $A^{a,S} \in \Sigma$ as follows: $A^{a,S}_i = \{a\}$ for $i \in S$, and $A^{a,S}_i = \emptyset$ for $i \in S^c$. For every v^a and for every $S \subseteq N$ we define

$$Q(v^a_S) = P(A^{a,S}).$$

We proceed to show that Q is well-defined. That is that $P(A^{a,S}) = P(A^{b,T})$ whenever $v^a_S = v^b_T$. We prove this with an induction on the size of S. Assume

$|S| = s$, $s \geq 1$, and the claim was proved for smaller sizes. Note that because $v^a_S = v^b_T$, for $i \in S$, $u_i(A^{a,S}) = \tau v^a_S(i) = \tau v^b_T(i) = u_i(A^{b,T})$. Because P is a potential for Γ,

$$u_i(A^{a,s}) = u_i(A^{a,S}) - 0 = u_i(A^{a,S}) - u_i(v^{a,S\setminus i}) = P(A^{a,S}) - P(v^{a,S\setminus i}),$$

and similarly:

$$u_i(A^{b,T}) = u_i(A^{b,T}) - 0 = u_i(A^{b,T}) - u_i(v^{b,T\setminus i}) = P(A^{b,T}) - P(v^{b,T\setminus i}).$$

By the induction hypothesis, (and because $v^a_{S\setminus i} = v^b_{T\setminus i}$, $P(v^{a,S\setminus i}) = P(v^{b,T\setminus i})$. Therefore the result follows. Note that because of the ψ efficiency of τ Theorem 1 implies that $\tau = \psi$ on H. ∎

A solution τ on $G(N)$ is *efficient* if it is φ-efficient, where φ is the Shapley value. Equivalently, τ is efficient if $\tau v(N) = v(N)$ for every game v. Consequently we have:

Corollary 2 *Let N be a finite set of players, let M be a finite set of facilities, and let $\Sigma_i \subseteq 2^M$ for every $i \in N$, with $\Sigma_i \neq \emptyset$. Assume in additions that $\{a\} \in \Sigma_i$ for every $i \in N$ and for every $a \in M$.*

Let τ be an efficient solution on $G(N)$. Let $G = F(N, M, (\Sigma_i)_{i\in N}, \tau, (v^a)_{a\in M})$ be a τ-form. If Γ_{F_G} is a potential game, the restriction of τ to $H = \cup_{a\in M} G_{v^a}$ coincides with the restriction of the Shapley value to H.

Consequently, if $\Gamma_{F_{G(N,M,(\Sigma_i)_{i\in N},\tau,(v^a)_{a\in M})}}$ is a potential game for every choice of cooperative games $v^a \in G(N)$, $a \in M$, τ is the Shapley value.

4 Congestion Games and Combinatorial Auctions

4.1 Combinatorial auctions

In a combinatorial auction (see e.g., [4]) there is a seller, denoted by 0, who wishes to sell a finite set of items I, that are owned by her. There is a finite set of buyers B. Let Π be the set of all allocations of the goods. That is, every $\pi \in \Pi$ is an ordered partition of I, $\pi = (\pi^a)_{a \in B \cup \{0\}}$. A *valuation function* of buyer a is a function $v^a : 2^I \to \Re$, where $\Re$ denotes the set of real numbers, with the normalization $v^a(\emptyset) = 0$.

Let V^a be the set of all possible valuation functions of a (obviously $V^a = V^m$ for all $a, m \in B$), and let $V = \times_{a \in B} V^a$. We assume each buyer knows his valuation function only.

A *mechanism* $M = (X, d, c)$ for allocating the goods (so called a combinatorial auction) is defined by sets of messages X^a, one set for each buyer a, and by a pair (d, c) with $d : X \to \Pi$, and $c : X \to \Re^B$, where $X = \times_{a \in B} X^a$. d is called the allocation function and c the transfer function; if the buyers send the profile of messages $x \in X$, buyer a receives the set of goods $d^a(x)$ and pays $c^a(x)$ to the seller; his utility is $u^a(v^a, x) = v^a(d^a(x)) - c^a(x)$. Every combinatorial auction generates a game in informational form, which is a Bayesian game without the probabilistic data.

A *strategy* of a in this game is a function $b^a : V^a \to X^a$. There are several solution concepts that are common in games in informational form. The most common are ex post domination, ex post equilibrium (see e.g., [9, 11]), and regret minimizing equilibria, which was recently defined and analyzed in [12].

A mechanism (X, d, c) is called a *direct* mechanism if $X^a = V^a$ for every $a \in B$. That is, in a direct mechanism a buyer's message contains a full description of some valuation function.

For an allocation π and a profile of valuations v we denote by $S(v, \pi)$ the *total social surplus* of the buyers, that is

$$S(v, \pi) = \sum_{a \in B} v^a(\pi^a).$$

We also denote:

$$S_{max}(v) = \max_{\pi \in \Pi} S(v, \pi).$$

A mechanism is called socially optimal if $S(v, d(v)) = S_{max}(v)$ for every $v \in V$. Socially optimal mechanisms are considered to be desirable. However, the terminology is misleading because the mechanism maximizes the social surplus obtained in the declared valuation functions. It is really socially optimal if it is also truth telling, in the sense that rational agents submit their true valuation functions. The VCG (Vickrey-Clarke-Groves) mechanisms [36, 3, 7] are such mechanisms. One of the main difficulty in utilizing the VCG mechanism, and in fact, any socially optimal auction mechanism are the huge computational and communication complexities involved in determining (online) the optimal allocation, the transfers, and the communication of data. Tackling these problems has been a subject of an extensive research.

4.2 Strategic goods

Consider a combinatorial auction setting, in which the set of goods I is a set of service providers whose services are being purchased by the buyers. In such a case the valuation function v^a of buyer a can be also interpreted as a cooperative game on the set of players I. In such a setup, a combinatorial auction mechanism should also specify the private payoff of every "good" $i \in I$. As long as the allocation of goods is done by a central organizer the problem is reduced to a revenue allocation problem. However, when the goods have strategic freedom, they can choose the buyer. Hence, a natural (direct) combinatorial auction with strategic goods is a two-stage game. At the first stage every buyer a submits a valuation function v^a (not necessarily the true valuation function), and in the second stage every $i \in I$ chooses a buyer, say m, and receives $\tau v^m_{S^m}(i)$, where S_m is the set of $j \in I$ that choose m, and $\tau : G(I) \to \Re^I$ is a solution for cooperative games. Therefore, every subgame of the two-stage combinatorial auction with strategic goods is a solution-based game as discussed in the previous section. Hence, if the solution concept is a semivalue, Theorem 2 implies that all subgames are potential games. In particular every subgame possesses a pure-strategy Nash equilibrium. Consider the following example, in which the solution is $\psi : G(I) \to \Re^I$, where

$$\psi v(i) = v(I) - v(I \setminus i), \quad \text{for every } v \in G(I), \text{ and for every } i \in I.$$

It can be easily verified that ψ is a semivalue. The potential function Q associated to ψ by Theorem 2 is defined by: $Qv = v(N)$. This function Q gives us the potential function P for the combinatorial auction, which is defined as follows:

$$P(m) = P((m_i)_{i \in I}) = \sum_{a \in B} Q(v^a_{S_a(m)}),$$

where $S_a(m) = \{i \in I | m_i = a\}$. It follows that an equilibrium of the potential game is obtained by maximizing the social surplus over all partitions.[13] The equilibrium assumption in economic theory asserts that in

[13] A potential game may have equilibrium profiles that do not maximize the potential. However, it was conjectured in [21] that players in real life games do have a tendency to

economic environments players choices are in equilibrium. The assumption seems to be more reasonable in games for which learning processes converge to equilibrium, such as potential games. Hence, if we take the equilibrium assumption seriously, letting the goods choose the allocation solves a computational problem, which is too complex for computers.[14] We therefore suggest that combinatorial auctions with utility maximizing goods may yield better social solutions if we give the goods a strategic freedom, rather than using centralized auctions. This is consistent with the recent success of information markets initiated in the Iowa electronic markets[15], which make prediction on political markets, and on economic indicator markets based on actions of many players.

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converge to an equilibrium profile that maximizes the potential.

[14] We do not know how do economic agents reach an equilibrium. Had we known it we could have programmed it.

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