

TO COMMIT or NOT TO COMMIT ?

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1. Introduction. The question of whether an economic agent can precommit himself has been extensively discussed in economic literature. There seems to be an implicit understanding (sometimes an explicit one, e.g., Scotchmer(1987)) that it is always better to commit, and the only question is whether a credible commitment can be made. For example, Rosenthal (1991) defines an equilibrium refinement concept, “the commitment-robust equilibria”: An equilibrium profile is “reasonable” if it is attained as a subgame perfect equilibrium outcome in each of the two extensive form games in which one of the players commits himself in the first stage to a mixed strategy. The option not to commit is not feasible in Rosenthal’s games, implicitly meaning that if a player can commit he will do so.

The goal of this note is to demonstrate that if a rational player has the option to precommit, he may choose not to do so. We model the commitment problem by extending the original game by introducing a new (first) stage in which one of the players, say Player 1, chooses one of two actions: to commit or not to commit.

We provide three examples that illustrate the three possible solutions to the commitment problem¹. In the first example, Player 1 chooses not to commit in the unique subgame

¹For an example to a tax evasion problem in which the commitment problem is not decisive, the reader is referred to Landsberger and *et al.* (1993).

perfect equilibrium of the commitment game that survives the process of sequential elimination of weakly dominated strategies. In the second example, Player 1 commits himself in every such subgame perfect equilibrium. Co-existence of commitment and no commitment is illustrated in the third example.

The use of successive weak dominance as a solution concept may be a controversial issue. This is partially due to the fact that there are examples showing that a strategy may be eliminated in a certain round only because of the presence of another strategy which in turn is eliminated in a future round (see e.g., Section 12.2 in Kreps (1990)). Such a phenomenon does not occur in our examples. Another objection to successive weak dominance is based on the high level of rationality and of common knowledge of rationality of players assumed implicitly. This objection makes sense only when the number of iterations required by the process is relatively big, which does not happen in our examples.

Our model allows commitments to mixed strategies, which makes the commitment option more desirable and therefore makes our result more striking. If only commitment to pure strategies are credible, then a player who can commit behaves exactly as a player that has the option to move first. Fudenberg and Tirole (1991) noted that a player in the Matching Pennies game prefers not to move first. Moving first gives him the payoff of -1, while in the simultaneous game he receives 0 in the unique equilibrium of the game (which is obtained in strictly mixed strategies). If however, one of the players in the Matching Pennies game has the option of committing to a mixed strategy, then this player becomes indifferent between committing or not committing himself. A commitment to a mixed strategy is quite natural in certain situations. E.g., a tax collecting agency may consider a commitment to

a randomized audit policy (“audit ten percent of the tax payers”).

2. The Commitment Game. Let $G = (X, Y, u^1, u^2)$ be a two-person game in strategic form. Player 1 chooses $x \in X$, Player 2 chooses $y \in Y$. Player 1 receives $u^1(x, y)$ and Player 2 receives $u^2(x, y)$. We assume that either X and Y are finite, or that they are compact metric spaces and the payoff functions are continuous. The set of mixed actions of the players are denoted by $\Delta(X)$ and $\Delta(Y)$ respectively. We identify X and Y as subsets of $\Delta(X)$ and $\Delta(Y)$ respectively, and we refer to members of X, Y as pure actions. We denote by $u^1(p, q)$ the expected payoff of Player 1 when he uses the mixed action p and Player 2 uses the mixed action q .

With each game G as described above, we associate a multistage game, denoted by CG (the commitment game): In stage 1 of CG , Player 1 has to choose one of two actions: commit or not commit. If he chooses not to commit, then the two players are engaged in playing G . If he chooses to commit, then in stage 2 he chooses a commitment strategy $p \in \Delta(X)$, and in stage 3, Player 2 (who is told about the commitment strategy p) chooses $y \in Y$. In case of commitment, the payoffs are determined as in G by u^i , $i = 1, 2$. As noted by Kuhn (1953), we can restrict ourselves to behavioral strategies in CG .

The following lemma illustrates that the decision to commit is always rational in the sense of being supported by a subgame perfect equilibrium in the commitment game. However, as shown in Example 2.1, such subgame perfect equilibria do not necessarily survive the process of elimination of weakly dominated strategies.

LEMMA 1. *There exists a subgame perfect equilibrium in CG , in which Player 1 commits himself.*

PROOF: We define strategies f for Player 1 and g for Player 2 as follows:

Let p^* be a commitment strategy, computed according to the *best* case scenario. That is,

$$(2.1) \quad p^* \in \operatorname{argmax}_{p \in \Delta(X)} \left(\max_{y \in BR(p)} u^1(p, y) \right),$$

where $BR(p)$ is the set of all pure best replies of Player 2 versus p .²

Let $y^* \in BR(p^*)$ be such that $u^1(p^*, y^*) \geq u^1(p^*, y)$ for every $y \in BR(p^*)$, and let (\bar{p}, \bar{q}) be any mixed strategy equilibrium in G .

In f Player 1 chooses to commit in the first stage, he chooses p^* in the commitment node, and he plays \bar{p} in the node of no commitment. In g , Player 2 reacts by y^* to a commitment to p^* , by an arbitrary best reply to any other commitment $p \in \Delta(X)$, and by \bar{q} to no commitment. It can be easily verified that (f, g) is a subgame perfect equilibrium in CG . ■

In the following example, the subgame perfect equilibrium described in the proof of Lemma 1 does not survive the process of sequential elimination of weakly dominated strategies in the game CG . Moreover, there exists a unique subgame perfect equilibrium that survives the elimination process. In this equilibrium Player 1 chooses not to commit.

EXAMPLE 2.1.

Consider the game G_1 , described below. The rows are labeled by x_1 and x_2 , and the columns are labeled by y_1 and y_2 .

$$G_1 = \begin{pmatrix} (9, 9) & (0, 9) \\ (5, 7) & (5, 6) \end{pmatrix}.$$

²It can be easily proved that such a strategy p^* must exist. That is, the argmax set, defined in (2.1) is not empty. This is in contrast to a commitment strategy computed under the worst case scenario, which need not exist even when G is a finite game.

We show that in any subgame perfect equilibrium that survives the process of sequential elimination of weakly dominated strategies, Player 1 does not commit himself.

As y_2 is weakly dominated by y_1 in G_1 , every strategy g of Player 2 in which he plays y_2 with a positive probability in the no commitment node is weakly dominated by the strategy g^* obtained from g by changing the behavior of g in the no commitment node to y_1 . Let g be a strategy of Player 2 in which he plays y_1 in the node of no commitment. It is easy to verify that g is weakly dominated if and only if there exists $p \in \Delta(X)$ such that the reaction in g to a commitment to p is not a best reply versus p . Note that there do not exist weakly dominated strategies for Player 1. We proceed to describe the second round of the elimination process. Let A denotes the set of all strategies of Player 2 that survive the first round of elimination.

It is obvious that every strategy f of Player 1 that calls with a positive probability for a commitment in the first move is weakly dominated with respect to A by f^* , where in f^* , Player 1 does not commit in the first move, plays x_1 in the no commitment node, and mimics f in the commitment node. Let f be a strategy of Player 1 in which he chooses in the first move not to commit. Let $p = (p(x_1), p(x_2))$ be the commitment strategy recommended by f . If $p(x_2) > 0$, then f is weakly dominated with respect to A by f^* , where f^* is obtained from f by changing the commitment strategy p to $p^* = (p(x_1) + \varepsilon, p(x_2) - \varepsilon)$ for sufficiently small $\varepsilon > 0$. Hence, there exists a unique strategy \bar{f} of Player 1 that survives the second round of elimination. In \bar{f} , Player 1 chooses not to commit, he plays x_1 in the no commitment node, and he chooses x_1 in the commitment node. The elimination process terminates after the second round. There exists a unique

$\bar{g} \in A$ that forms a subgame perfect equilibrium with \bar{f} . In \bar{g} , Player 2 plays y_1 in the no commitment node, and reacts by y_1 to every commitment strategy $p \in \Delta(X)$.

In the following example Player 1 commits himself in every subgame perfect equilibrium of the commitment game.

EXAMPLE 2.2.

Consider the game G_2 ,

$$G_2 = \begin{pmatrix} (1, 1) & (5, 0) \\ (0, 0) & (4, 4) \end{pmatrix}.$$

In every subgame perfect equilibrium of the commitment game Player 1 commits to x_2 and plays x_1 in the no commitment node. Player 2 plays y_1 in the no commitment node and reacts by y_2 to the commitment of x_2 . Actually, all subgame perfect equilibria in the commitment game differ only in the behavior of player 2 in the node where Player 1 commits to the mixed strategy $p = (0.8, 0.2)$.

The next example is a version the Matching Pennies game. We show that the commitment problem is not decisive in this game.

EXAMPLE 2.3.

Consider the game G_3 ,

$$G_3 = \begin{pmatrix} (1, 0) & (0, 1) \\ (0, 1) & (1, 0) \end{pmatrix}.$$

In the associated commitment game there exist many subgame perfect equilibria indexed by two parameters $0 \leq \alpha, \beta \leq 1$. All those equilibria survive the elimination process. In each such subgame perfect equilibrium, player 1 chooses to commit with a probability α , and chooses $(0.5, 0.5)$ in both the commitment node and the no commitment node.

Player 2 chooses $(0.5, 0.5)$ in the node of no commitment, chooses the unique best reply versus a commitment to $p \neq (0.5, 0.5)$, and chooses $q = (\beta, 1 - \beta)$ versus a commitment to $p = (0.5, 0.5)$.

For illustration let us analyze the commitment game associated with G_1 without applying the process of successive elimination of weakly dominated strategies. At the node of commitment there is a unique equilibrium. In this equilibrium, Player 1 commits to x_1 and Player 2 reacts by y_1 . This equilibrium yields a payoff of 9 to Player 1. At the node of no commitment there are many equilibria indexed by $\frac{5}{9} \leq \beta \leq 1$. In each such equilibrium Player 1 plays x_1 and Player 2 plays the mixed action $(\beta, 1 - \beta)$, and the payoff to Player 1 is 9β . Consequently, Player 1 who believes in subgame perfection chooses to commit in CG_1 .

3. Other Approaches. Modeling the commitment problem as a game and analyzing it by applying equilibrium theory to this game is by no means the only reasonable approach to this problem. A commitment strategy may be chosen as a security level strategy; The committing player assumes that his rival will choose the worst (from the committing player point of view) best reply versus the commitment strategy. He then chooses his commitment strategy in order to maximize his security level. That is, for each mixed strategy $p \in \Delta(X)$, let

$$v(p) = \min_{q \in BR(p)} u^1(p, q),$$

and let $v = \sup_{p \in \Delta(X)} v(p)$. We call v the *commitment value* for Player 1. p^* is a commitment strategy for player 1 if and only if, $v(p^*) = v$. If a commitment strategy does not exist (which may happen even if we deal with finite games), we can use ε -commitment

strategies: p is an ε -commitment strategy if $v(p) > v - \varepsilon$. Consider (again) the game G_1 . It can be easily verified that the commitment value of this game equals 9. A commitment strategy does not exist, but by committing to $p = (1 - \varepsilon, \varepsilon)$ Player 1 can guarantee a payoff of $9(1 - \varepsilon)$. Hence, Player 1 can guarantee himself $9(1 - \varepsilon)$ by choosing to commit under the assumption that his rival is rational enough to always use a best reply. He can guarantee himself 9 by choosing not to commit under the assumption that his rival is sufficiently rational in the sense of not using weakly dominated strategies. Hence, as before, the final result depends on the attitude of players to weakly dominated strategies.

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