Change of Stochastic Parameters in Derivative Assets

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Abstract

It can be shown that under the assumptions of the Black-Scholes model, option prices are functions of moneyness and time to maturity (ttm). On the other hand, these prices are very complicated functions of the fundamental parameters of strike and expiration date.

It is a common practice to quote option prices in terms of their moneyness and time to maturity (relative parameters) instead of strike and expiration date (absolute parameters). One of the reasons for using these parameters is the fact that in the setting of the Black-Scholes model, the price of an option, expressed in units of the underlier and parametrized by moneyness and time to maturity, is independent of time and stock prices. For this reason, in some instances both academics and practitioners find it more appealing to specify prices in terms of alternative parameters. Similarly defined relative parameters are used in the study of the term structure of interest rates and of the surface of stock options implied volatility.

We focus on developing tools for analyzing derivative assets under alternative parametrizations, and demonstrating their application in the case of options quoted in terms of moneyness and ttm, rather than the standard strike and maturity date.

According to the fundamental theorem of asset pricing (FTAP) if there is no arbitrage in the market, there exists a probability measure (called the risk neutral measure) so that asset prices are martingales with respect to this measure. However, alternatively parametrized assets are not traded in the market and therefore their prices are not necessarily martingales under the risk neutral measure.

We show that under the assumption that asset prices, specified in terms of alternative parameters, are arbitrage free there exists an equivalent probability measure under which these prices are martingales. This result allows us to apply the useful properties of martingales in order to price various types of derivatives, whose payoffs are functions of moneyness and ttm, in a similar manner to how it’s done under the FTAP.