Stochastic Enumeration Methods for Counting, Rare-Events and Optimization

Radislav Vaisman - PhD candidate
Supervisor: Professor Ofer Strichman

Faculty of Industrial Engineering and Management, Technion.

Abstract
Monte Carlo methods present a class of computational algorithms that rely on repeated random sampling to approximate some unknown quantities. They are most suited for calculation using a computer program and they are typically used when the exact results with a deterministic algorithm are not available.

The \#P complexity class, introduced by Valiant (Valiant 1979), consists of the set of counting problems that are associated with decision problems in NP, e.g., how many solutions does a propositional formula has (\#SAT). The \#P-complete complexity class is a sub-class of \#P, consisting of those problems in \#P that any other problem in \#P can be reduced to them via a polynomial reduction. (\#SAT), for example, is also \#P-complete. Interestingly various \#P-complete problems correspond to easy decision problems, i.e., in P, such as satisfiability of propositional formulas in Disjunctive Normal Form (DNF).

For some \#P-complete problems, there are known efficient approximations. For example, Karp and Lubby (Karp and Luby 1983) introduced a fully polynomial randomized approximation scheme (FPRAS) for counting the solutions of disjunctive normal form (DNF) satisfiability formulas. Similar results were obtained for the knapsack and permanent counting problems (Dyer 2003, Jerrum et al. 2004). Most of those algorithms uses a Monte Carlo procedures that operates under rare-event settings.

We present a state of the art approaches to handle hard counting problems. In particular the Markov Chain Monte Carlo and Importance Sampling. We also show that the classical Spectra technique that is widely used in Network Reliability can be adapted for counting monotone CNF formulas. To the best of our knowledge, this is the first rigorous result available for the Spectra method.