Rethinking Biased Estimation: Improving Maximum Likelihood and the Cramer-Rao Bound

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One of the goals of statistical estimation theory is the development of performance bounds when estimating parameters of interest in a given model, as well as determining estimators that achieve these bounds. When the parameters to be estimated are deterministic, a popular approach is to restrict attention to unbiased estimators and develop bounds on the smallest mean-squared error (MSE) achievable within estimators of this class. Although it is well-known that lower MSE can be achieved by allowing for a bias, in applications it is typically unclear how to choose such an appropriate bias.

In this talk we develop bounds that dominate the conventional unbiased Cramer-Rao bound (CRB) so that the resulting MSE bound is lower than the CRB for all values of the unknowns. When an efficient maximum-likelihood (ML) estimator achieving the CRB exists, we show how to construct an estimator with lower MSE regardless of the true unknown values, by linearly transforming the ML estimator.

We then specialize the results to linear estimation in linear models. In particular, we derive a class of estimators with lower MSE than the conventional least-squares approach for all values of the unknown parameters, thus leading to estimation methods that are provably better than least-squares.

The procedures we develop are based on a saddle-point formulation of the problem which admits the use of convex optimization tools.